Instructions: Show all your work for full credit, and box your answers when appropriate.

1. Mike deposits $100 at the end of every 6 months into an account for 30 years. (The last payment occurs in 30 years.) The account earns a nominal annual interest of 5% compounded semi-annually. Mia makes 10 level payments of X into an account every 2 years with her last payment occurring at the same time as Mike’s last payment. If Mia has twice as much as Mike right at their last deposit, compute X. The answer is 1626, 1676, 1726, 1776, 1826.

Solution: Twice the future value of Mike’s account is

$$2 \times 100 \times s_{60;2.5} = 27198.$$  

Mia’s account earns $$(1 + 0.025)^4 - 1 = 10.38\%$$ every two years so her future value is $$Xs_{10;10,38}$$. Set equal and solve for $$X = 1676$$.

2. Your investment project requires an initial deposit of 1 million. It pays 200,000 a year at the end of the year for three years, then it pays 150,000 at the end of the year for 4 more years after that.

(a) Compute the internal rate of return.

The answer is 2.12, 3.12, 4.12, 5.12, 6.12, 7.12%

Solution: In CF menu, plug in $$CF_0 = -1000000, C01 = 200000, F01 = 3, C02 = 150000, F02 = 4$$ and compute $$IRR = 5.12$$.

(b) Suppose the net present value of the project is 50,000. In what one-percentage range does the effective annual yield lie? 0 to 1%, 1 to 2%, 2 to 3%, ..., n to (n + 1)%, ...? (So for example, if the yield was 17.25 your answer would be the 17 to 18% range.)

Solution: Method 1: guess different I’s in NPV menu and see that it is above 50000 for $$I = 4$$ and below 50000 for $$I = 3$$.

Method 2: change $$CF_0 = -1050000$$ and recompute $$IRR = 3.7$$. A common mistake: setting $$CF_0 = -950000$$.

3. You own a 20-year annuity immediate with annual payments of 50. You own a 20-year 1000 face value (which we assume equals the redemption value) bond which pays 8% coupons annually. Assume that all effective annual spot rates are 4%. By how many basis points should these rates (simultaneously) change for your portfolio...
to lose 5% of its present value? Use the modified duration of your combined annuity- 
bond portfolio to estimate your answer.

The answer is 25, 35, 45, 55, 65, -25, -35, -45, -55, -65.

Solution: You could compute the modified durations of the bond and annuities 
individually and then take their weighted sum, or treat them as a single asset, and 
compute its modified duration. Let us do latter: if you receive \( 50 + 80 = 130 \) 
every year then you can imagine you have a 20-year 1000 face value bond with 13% 
coupons annuals. The formula for its duration is 
\[
D = \frac{(50 + 80) \nu_{20} + 20 \times 1000 \times 1.04^{-20}}{130 \nu_{20} + 1000 \times 1.04^{-20}} = 11.4
\]
years. So the modified duration is \( DM = 11.4/1.04 = 10.98 \) years We want to find 
a change in interest rate \( \Delta i \) which induces a 5% decrease in price, \( \Delta P/P = -0.05 \).

\[
-0.05 = \Delta P/P = -DM\Delta i = -10.98\Delta i
\]
which implies \( \Delta i = 0.0045 \), or 45 basis points.

4. A one-year zero-coupon bond with face value 50 is priced today at 47. A three-
year zero-coupon bond with face value 100 is priced today at 87. The forward rate 
from year 1 to year 2 is 4% compounded annually. At what rate should a two-year 
bond bear annual coupons in order for its face value (which we assume equals the 
redemption value) to be equal to its price.

No answer given since both a correct and incorrect approach give a similar answer.

Solution: I can think of three approaches to this problem. Two of them involve 
swaps. Recall the par rate (the answer we are seeking) is the same as the swap rate 
\( R \) on a two-year swap. Let \( i_{n-1,n} \) denote the forward rate and \( s_n \) denote the spot 
rate (annual compounding for both) and \( P(0, n) = (1 + s_n)^{-n} \) denote the price of a 
n-year 1 face 0-coupon bond. Then there are two formulas for the swap rate \( R \):

\[
R = \frac{P(0, 1)i_{0,1} + P(0, 2)i_{1,2}}{P(0, 1) + P(0, 2)} \quad \text{or} \quad R = \frac{1 - P(0, 2)}{P(0, 1) + P(0, 2)}.
\]

The other approach is to work directly with bonds. We know the PV of a 2-year 
R-coupon bond is its face value; thus, setting the PV and face both equal to 17 (or 
whatever), we get,

\[
17 = 17R(1 + s_1)^{-1} + 17R(1 + s_2)^{-2} + 17(1 + s_2)^{-2}.
\]

Let us solve all the necessary rates and prices (note the 3-year bond is a red herring). 
\( P(0, 1) = 47/50 \) so \( i_{0,1} \) (which is the same as \( s_1 \)) is \( 50/47 - 1 = 6.38\% \). We already 
have \( i_{1,2} = 0.04 \), and \( (1 + s_2)^2 = (1 + 0.0638)(1 + 0.04) = 1.1064 \) so \( P(0, 2) = 
1/1.1064 = 0.9038 \).
Note we do not need to solve $s_2 = 5.19\%$ explicitly, although many did that and claimed $R = s_2$ as an answer, which it is not. After using one of the three approaches, you get $R = 5.22\%$.

5. The one year spot rate is 4\% per annum with continuous compounding. Google currently trades for 600. It is not expected to pay any dividends over the next year. Assume there are no arbitrage opportunities. Let $p_K$ denote the price (premium) of a one-year European put option on Google with strike $K$. You know that

$$p_{580} = 4, p_{590} = 6, p_{600} = 9, p_{610} = 13, p_{620} = 20.$$ 

Suppose you write a strangle with strikes 590 and 610. (That is, the payoff function changes slopes at 590 and 610.) For what range of Google stock prices in one year will your profit be positive?

No answer given.

**Solution:** A written strangle is short the put with strike 590 and short the call with strike 610. The premium for this is

$$p_{590} + c_{610} = p_{590} + p_{610} + 600 - 610e^{-0.04} = 32.91$$

Several people wrote down $p_{590} + p_{610}$ instead. The future value of this premium (another common omission) is $32.91e^{0.04} = 34.25$. The written strangle is thus profitable in the range $(590 - 34.25, 610 + 34.25) = (555.75, 645.25)$. This is perhaps easiest to see by graphing the payoff and then shifting vertically up by 34.25 to get the profit.