Solutions to practice midterm if options are not covered
(there may be numerical mistakes)

1. Consider Bond A which is an 8%-coupon bearing bond maturing in 15 months, with principal $100. Coupons are paid semi-annually. Suppose (continuously compounding) spot rates are the following: 3 month is 5.0% 6 month is 5.1% 9 month is 5.2% 12 month is 5.3% 15 month is 5.4%.

   (a) Compute the equivalent rates with quarterly compounding.

   (b) Compute the yield and duration of the Bond A.

   (c) Suppose the Fed raises rates by 25 basis points. Use the duration to estimate the new price of the Bond A.

   (d) Suppose a bond maturing in 24 months with principal $100 is priced at $88. Compute the forward rate between months 6 and months 24.

   Solution:

   The bond price is

$$B = 4e^{-0.05*3/12} + 4e^{-0.052*9/12} + 104e^{-0.054*15/12} = 105.00$$

   The bond yield y solves

$$4e^{-y*3/12} + 4e^{-y*9/12} + 104e^{-y*15/12} = 105.00$$

   which implies $y = 0.0539$. Note that it is close to 5.4% The duration (in years) is then

$$D = \frac{3/12 * 4e^{-0.0539*3/12} + 9/12 * 4e^{-0.0539*9/12} + 15/12 * 104e^{-0.0539*15/12}}{105} = 1.18$$

   Note that it is close but just below 1.25 years.

   The change is bond price is approximately

$$\Delta B = -DB\Delta y = -1.18 * 105 * 0.0025 = -0.30$$

   So the new price is approximately 104.70

   The 2-yrs zero rate r_2 is given by

$$88 = 100e^{-r_2*2}, \quad \text{or } r_2 = \frac{-\ln(88/100)}{2}$$

   The forward rate $r_{0.5,2}$ is then gotten from this and the 6-month rate $r_{0.5} = 0.051$ from the relation

$$r_{0.5} * 0.5 + r_{0.5,2} * 1.5 = r_2 * 2$$

2. The spot price of Google stock is \$400. The stock will pay a \$10 dividend in four months and again in 7 months. The forward price of one share Goggle with delivery date in 9 months is \$600. The risk-free rate is 5% per annum with continuous compounding. Construct an arbitrage opportunity involving one Google share. What is the profit from this opportunity?

   Solution:

   Since the forward price is too high

   $$600 > (400 - 10e^{-0.05*4/12} - 10e^{0.05*7/12})e^{0.05*9/12}$$

   you want to take a short position in the forward now and borrow money to buy the stock. You repay the loan and deliver the stock for the delivery price. Note the stock will pay you dividends of \$10 in months 4 and 7. So you borrow $10e^{-0.05*4/12}$ for 4 months, $10e^{-0.05*7/12}$ for 7 months and the remaining $400 - 10e^{-0.05*4/12} - 0e^{-0.05*7/12}$ for 9 months.

   The present value profit is

   $$600e^{-0.05*9/12} - 400 - 10e^{-0.05*4/12} - 10e^{0.05*7/12}.$$

3. A bank has entered a swap where it pays the floating 6-month LIBOR rate (semi-annual compounding) and receives a fixed 8% per annum (semi-annual compounding). The notional principal is \$10 million. Coupons are exchanged every six months. The swap expires in 9 months. Current LIBOR rates on all maturities are 5% per annum with continuous compounding. The 6-month LIBOR rate 3 months ago was 6% per annum (semi-annual compounding). Compute the value of the swap to the bank.

   Solution:

   Value of swap is value of fixed minus value of float. Value of fixed is (in millions)

   $$.4e^{-.05*3/12} + 10.4e^{-.05*9/12} = 10.41$$

   Value of float is (in millions)

   $$(10 + .3)e^{-.05*3/12} = 10.17$$

   So swap worth 240,000.

4. Today (March 27) Cisco stock currently costs \$20. The stock is will pay of dividend of \$1 in 1 months and in 4 months. The risk-free rate is 5% per annum with continuous compounding.

   (a) Compute the forward price of Cisco stocks with delivery date in 9 months (December 27).

(b) Two months ago (January 27) you had taken a long position in a forward contract with deliveray date December 27 and delivery price \$20. What is the value of your position today (March 27)?

Solution:

The present value of cash flows is

$$D = 1e^{-.05*1/12} + 1e^{-.05*4/12} = 1.98.$$

So forward price is

$$F = (20 - 1.98)e^{.05*9/12} = 18.71$$

Note if you use $S - Fe^{-rT}$ be sure to use effective stock price rather than actual stock price. Alternatively, value is

$$(18.71 - 20)e^{-.05*9/12} = -1.24$$

5. Consider the following bonds, each with principal 100 and paying 5% per annum coupons every six months. The bond that matures in 6 months costs 98, the bond that matures in 9 months costs 99, the bond that matures in 12 months costs 97, the bond that matures in 15 months costs 100. Deduce the 15-month spot rate.

   Solution: This problem is incomplete in that you need to know the 3 month rate (or the price of a bond maturing in 3 months without any coupons). Note the 6 month and 12 month bonds are irrelevant to the problem.