Math 441, Solutions to Midterm Fall 2005

1. Futures price is

\[ F = 16(400 + 5)(1 + 0.006/4)^2 = 6676. \]

2. Hedge ratio is

\[ h = \frac{\rho \sigma_S}{\sigma_F} = (0.08) \ast (0.25)/0.20 = 1. \]


3. (a) 

\[ SD(\Delta X) = \sqrt{E(\Delta X^2) - E(\Delta X)^2} \]

\[ = \sqrt{(0.45 \ast 10^2 + 0.55 \ast (-5)^2) - (0.45 \ast 10 + 0.55 \ast (-5))} \]

\[ = 7.60 \]

(b) Expected profit from SHORT position is 

\[ (0.45) \ast (75 - 80) + (0.55) \ast (75 - 65) = 3.25 \]

4. (a) Let \( r_1 \) and \( r_2 \) be the one-year and two-year spot rates.

\[ 97 = 100 \exp\{-r_1\} \]

implies \( r_1 = 3.0\% \).

\[ 100 = 4 \exp\{-0.030\} + 104 \exp\{-2r_2\} \]

implies \( r_2 = 3.9\% \). The forward rate is \( (2 \ast 0.039 - 1 \ast 0.030)/(2 - 1) = 4.8\% \).

(b) Let \( y \) be the yield. It solves

\[ (2 \ast 97 + 100) = (2 \ast 100 + 4)e^{-y} + 104e^{-2y} \]

ie, \( y = 3.5\% \), Duration and convexity are

\[ D = \frac{204e^{-0.035} + 2 \ast 104e^{-0.035 \ast 2}}{297} = 1.316 \]

\[ C = \frac{204e^{-0.035} + 4 \ast 104e^{-0.035 \ast 2}}{297} = 1.99. \]

Let \( \Delta y = 0.025 \) be yield change, let \( B, B' \) be old and new price resp.

\[ B' \approx B + (-DB\Delta y) + \frac{1}{2}CB(\Delta y)^2 = \$284.50. \]
5. The total gain is \((5.4 - 4.4) - (7.0 - 6.8) = 80\) basis points. 30 for (intermediary Japanese) bank, 25 each for A and B. B borrows US from outside paying 4.4 US. So it must receive 4.4 US from bank. It therefore pays bank \(6.9 - 0.025 = 6.55\) yen. Bank wants gain in yen only so pays A 6.25 yen and receives 4.4 US. A borrows externally 7.0 yen. Check: A pays 7.0 - 6.25 yen and 4.4 US which is 5.4 - 0.025.

Physically, A pays \((0.75)/2 \times 1,200,000,000 = 4,500,000\) yen.

6. \$1500 is obviously too high. So today, Lex takes short position, borrows \$1000, buys 1 pound krypto. In a year, he delivers krypto and pays back loan for profit of \(1500 - 1000(1 + 0.06/2)^2 = 439.10\). In today’s dollars, that’s \(439.1(1 + 0.06/2)^{-2} = \$413.89\).