

## Midterm

1. Assume all spot rates are 5% per annum with continuous compounding.

(a) Compute the equivalent rate with quarterly compounding.

$$e^{.05} = (1 + r/4)^4 \rightarrow r = 0.0503$$

(b) Consider a 8%-coupon bearing bond with principal \$120 which expires in 15 months. Coupons are paid semi-annually. Compute the price and yield of the bond.

Yield is 5% and price is

$$4.8e^{-.05*3/12} + 4.8e^{-.05*9/12} + 124.8e^{-.05*15/12} = 126.60$$

2. A bank has entered a swap where it pays the floating 6-month LIBOR rate and receives a fixed 8% per annum. The notional principal is \$10 million. Coupons are exchanged every six months. The swap expires in 9 months. Current LIBOR rates on all maturities are 5% per annum with continuous compounding. The 6-month LIBOR rate 3 months ago was 6% per annum. Compute the value of the swap to the bank.

Value of swap is value of fixed minus value of float. Value of fixed is (in millions)

$$.4e^{-.05*3/12} + 10.4e^{-.05*9/12} = 10.41$$

Value of float is (in millions)

$$(10 + .3)e^{-.05*3/12} = 10.17$$

So swap worth 240,000.

3. This problem continues on the next page. Suppose Company A want to borrow 110 million Yen and Company B want to borrow 1 million USD (US Dollars). The current exchange rate is 1 USD to 110 yen.

- A can borrow USD at 5% per annum with semi-annual compounding
- A can borrow Yen at 6% per annum with semi-annual compounding
- B can borrow USD at 6.2% per annum with semi-annual compounding
- A can borrow Yen at 6.5% per annum with semi-annual compounding

- (a) Construct a swap, with an intermediary bank netting 30 basis points. Make it so that the bank bears all the risk associated to changing foreign exchange rates and that the swap appears mutually attractive to A and B.

A and B each can save 20 basis points: A borrows externally at 5 USD; B borrows externally at 6.5 Yen; A gives bank 5.8 Yen and receives 5 USD; B gives bank 6 USD and receives 6.5 Yen

- (b) Determine the exchange rate for which the bank neither gains nor loses at each coupon payment.

Let  $k$  Yen to 1 USD be the future exchange rate. The bank nets zero when when  $6\% - 5\%$  of 1 million USD equals  $6.5\% - 5.8\%$  of 110 million Yen times  $k$  Yen per USD. Get  $k = 77.4$ .

4. Today (March 27) Cisco stock currently costs \$20. The stock is will pay of dividend of \$1 in 1 months and in 4 months. The risk-free rate is 5% per annum with continuous compounding. This problem continues on the next page.

- (a) Compute the futures price of Cisco stocks with delivery date in 9 months (December 27).

The present value of cash flows is

$$I = 1e^{-.05*1/12} + 1e^{-.05*4/12} = 1.98.$$

So futures price is

$$F = (20 - 1.98)e^{.05*9/12} = 18.71$$

- (b) Two months ago (January 27) you had taken a long position in a futures contract with delivery date December 27 and delivery price \$20. What is the value of your position today (March 27)?

Note if you use  $S - Fe^{-rT}$  be sure to use effective stock price rather than actual stock price. Alternatively, value is

$$(18.71 - 20)e^{-.05*9/12} = -1.24$$

5. A futures contract on S&P 500 has a contract size of 250. The spot price of S&P is \$1050. Suppose the risk-free rate is (and always has been) 5% per annum. You are a hedge fund manager. Last year when the S&P's annual return was 4% your portfolio's return was 2%. The portfolio is currently worth \$10 million. All rates of return are annually compounded.

- (a) What is the beta,  $\beta$ , of your portfolio?

$$\beta = \frac{0.02 - 0.05}{0.04 - 0.05} = 3$$

- (b) Suppose you want to reduce your  $\beta$  by half for (just) the next two months. (Because of upcoming political developments, you fear a highly volatile stock market over the next two months.) The next futures delivery date is in 3 months. What do you do?

$$(3 - 1.5) \frac{10,000,000}{1050 * 250} = 57.14$$

So short 57 futures. Close out position in two months.