

## Midterm

October 24, 2006

Name: \_\_\_\_\_

**Instructions:** Show all your work for full credit, and box your answers when appropriate. There are 5 questions: the first 4 are each worth 10 points and the last one is worth 15 points. Unless otherwise noted, all risk free rates are per annum with continuous compounding.

1. Company A wishes to borrow 10 million USD. Company B wishes to borrow 12 million AUD (Australian dollars). The current FX (foreign exchange) rate is 1 USD for 1.2 AUD. Company A is offered the following rates per annum: 10% USD, 8 % AUD. Company B is offered the following rates per annum: 9 % USD, 9.5% AUD.

Construct a swap where interest payments are exchanged semiannually, through an intermediary bank subject to the following conditions: The bank must bear all of the risk associated to fluctuating exchange rates; the bank must net a profit of 15 basis points at each exchange; the remaining (initial) savings are to be split evenly between A and B.

**Solution:**

The overall (annual) advantage is  $(10 - 8) - (9 - 9.5) = 2.5\%$  to be split between A and B after the bank takes 15 basis points (0.15%) at each exchange for a total of 0.3% annually. So A and B each have a 1.1% annual advantage. Thus A borrows AUD at 8% and enters into a swap, swapping a 8% AUD income stream for a  $10 - 1.1 = 8.9\%$  USD loan with the bank. And B borrows USD at 9% and enters into a swap, swapping a 9% USD income stream for a  $9.5 - 1.1 = 8.4\%$  AUD loan with the bank.

2. Consider Bond A which is an 8%-coupon bearing bond maturing in 15 months, with principal \$100. Coupons are paid semiannually. Suppose (continuously compounding) spot rates are the following: 3 month is 5.0%, 6 month is 5.1%, 9 month is 5.2%, 12 month is 5.3%, and 15 month is 5.4%.

- (a) Compute the price and the duration of Bond A.

**Solution:**

The bond price is

$$B = 4e^{-0.05 \cdot 3/12} + 4e^{-0.052 \cdot 9/12} + 104e^{-0.054 \cdot 15/12} = 105.00$$

The bond yield  $y$  solves

$$4e^{-y \cdot 3/12} + 4e^{-y \cdot 9/12} + 104e^{-y \cdot 15/12} = 105.00$$

which implies  $y = 0.0539$ . Note that it is close to 5.4% The duration (in years) is then

$$D = \frac{3/12 * 4e^{-0.0539 \cdot 3/12} + 9/12 * 4e^{-0.0539 \cdot 9/12} + 15/12 * 104e^{-0.0539 \cdot 15/12}}{105} = 1.18$$

Note that it is close but just below 1.25 years.

- (b) Suppose the Federal Reserve raises interest rates and this causes yields to increase by 25 basis points across all maturities. Use the duration to estimate the new price of Bond A.

**Solution:**

The change in bond price is approximately

$$\Delta B = -DB\Delta y = -1.18 * 105 * 0.0025 = -0.30$$

So the new price is approximately 104.70

3. The spot price of Google stock is \$420. The stock will pay a \$10 dividend in 2 months and again in 5 months. The futures price of one share of Google with delivery date in 6 months is \$450. You can invest money at a rate of 4.5% and borrow money at a rate of 5% with continuous compounding. Construct an arbitrage opportunity involving one Google share. What is the profit from this opportunity?

**Solution:**

You want to take a short position now and borrow money to buy the stock. You repay the loan and deliver the stock for the delivery price. There are two ways to do this.

Note the stock will pay you dividends of \$10 in months 2 and 5. This means you can either borrow the full \$420 for 6 months and invest the dividends at the 4.5% rate for  $6 - 2 = 4$  and  $6 - 5 = 1$  months respectively, or you can borrow  $10e^{-0.05*2/12} = 9.92$  for two months,  $10e^{-0.05*5/12} = 9.79$  for 5 months and  $420 - 9.92 - 9.79$  for 6 months.

First method: the profit (in 6 months) is  $450 + 10e^{0.045*(6/12-2/12)} + 10e^{0.045*(6/12-5/12)} - 420e^{0.05*6/12} = 39.56$ .

Second method: the profit (in 6 months) is  $450 - (420 - 9.92 - 9.79)e^{0.05*6/12} = 39.58$ .

4. Suppose a financial institution has agreed to pay 6-month LIBOR and receive 7% per annum (with semiannual compounding) on a principal amount of \$2 million. The swap has a remaining life of 1.75 years. The 6-month LIBOR rate 3 months ago was 4.5% (with semiannual compounding). Today, the LIBOR rates with continuous compounding for 3-month, 9-month, 15-month, and 21-month maturities are 4%, 4.4%, 4.8%, and 5.1%, respectively.

(a) What is the value of this swap today?

**Solution:**

The swap is long a fixed bond and short a floating bond. The fixed bond is worth (in thousands of dollars)

$$70e^{-(.04)(.25)} + 70e^{-(.044)(.75)} + 70e^{-(.048)(1.25)} + 2,700e^{-(.51)(1.75)} = 2,096.20$$

The floating bond is worth the present value of the principal plus the next interest payment:

$$2,045e^{-(.04)(.25)} = 2,024.70$$

The swap is thus worth about \$71,500.

- (b) Suppose a zero coupon bond maturing in 24 months with principal \$100 is priced at \$88. Compute the continuously compounded forward rate between month 9 and month 24.

**Solution:**

The 9-month zero rate is 4.4% from part (a). The 24 month is computable from this bond:  $88 = 100e^{-2r}$ , so  $r = 6.39\%$ . The forward rate is

$$\frac{(6.39)(2) - (4.4)(.75)}{1.25} = 7.59\%.$$

5. (15 points) The price of Kimberly-Clark stock (KMB) is currently \$65. It is predicted that at the end of each of the next 3 months, the price will be either \$1 higher or \$1 lower than the price at the start of the month. The probability that it will be \$1 higher is 60% and that it will be \$1 lower is 40%.

- (a) What is the expected value of the price of KMB in 3 months? What is the standard deviation of the price?

**Solution:**

Let  $X$  be the stock price in 3 months. There are 4 outcomes of stock price.

$$\begin{aligned}P(X = 62) &= 0.4^3 = 0.064, \\P(X = 64) &= 3 * 0.4^2 * 0.6 = 0.288, \\P(X = 66) &= 3 * 0.4 * 0.6^2 = 0.432, \\P(X = 68) &= 0.6^3 = 0.216.\end{aligned}$$

So  $E[X] = 0.064 * 62 + 0.288 * 64 + 0.432 * 66 + 0.216 * 68$ . And

$$SD(X) = \sqrt{E[X^2] - E[X]^2} = \sqrt{4306.24 - 4303.36} = 1.70$$

- (b) Suppose you own a portfolio consisting of 100 shares of KMB and a short position in a futures contract for 100 shares of KMB. The futures contract is for delivery in 3 months at a delivery price of \$70. What is the expected value of your portfolio in 3 months?

**Solution:**

Since you will deliver the stocks for \$70 each regardless, your (expected) value is  $100 * 70 = 7000$ .

- (c) Assume that the 3 month zero rate is 6% per annum (with continuous compounding). What is the value of the portfolio today assuming KMB does not pay a dividend.

**Solution:**

The present value of part (b), or alternatively, the value of the stock which is 6500 plus the value of the futures contract which is currently worth

$$7000e^{-(.06)(.25)} - 6500$$

(since you are short the contract), for a total of  $7000e^{-(.06)(.25)}$ .