Journal of Statistical Physics, Vol. 19, No. 2, 1978

The Statistics of Curie–Weiss Models

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Received November 29, 1977; revised February 22, 1978

Let S_n denote the random total magnetization of an n-site Curie–Weiss model, a collection of n (spin) random variables with an equal interaction of strength $1/n$ between each pair of spins. The asymptotic behavior for large n of the probability distribution of S_n is analyzed and related to the well-known (mean-field) thermodynamic properties of these models. One particular result is that at a type-k critical point $(S_n - nm)/n^{1-1/2k}$ has a limiting distribution with density proportional to $\exp[-\lambda s^{2k}/(2k)!]$, where m is the mean magnetization per site and λ is a positive critical parameter with a universal upper bound. Another result describes the asymptotic behavior relevant to metastability.

KEY WORDS: Block spin; renormalization group; mean-field; Curie–Weiss.

1. INTRODUCTION

The classical Curie–Weiss theory of magnetism occupies a central place in the physical literature. Based on the device of a self-consistent (or mean) field, the theory allows one to readily study the behavior of thermodynamic quantities such as specific heat, isothermal susceptibility, and magnetization in the neighborhood of the critical point. Unfortunately, the predictions of this classical theory do not completely agree with experiment, and so other theories, like nearest neighbor Ising models, must be considered. However, because of its relative simplicity and the qualitative correctness of at least some of its predictions (e.g., it works well away from the critical point), the Curie–Weiss theory has been historically important.[1,16]

Research supported in part by National Science Foundation Grants MPS 76-06644 (to RSE) and MPS 74-04870 A01 (to CMN).

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0022-4715/78/0800-0149$05.00/0