Statistical Theories of Turbulence and Applications to the Large-Scale Flows on Jupiter

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Research Objectives. Physical phenomena as complicated as fluid turbulence require a statistical formulation before they are amenable to scientific computation. Our goal is to use equilibrium statistical mechanics to describe the large-scale behavior of complex systems without computing the details of their small-scale dynamics. Geophysical fluid dynamics provides a wealth of examples having important applications to the Earth's oceans and atmosphere. Another prime example, and a test case for theory, is the atmosphere of Jupiter. In particular, we seek to explain the large-scale flows on Jupiter, which consist of alternating eastward and westward jetstreams, the Great Red Spot, and other vortices.

Scientific Approach. We use a synthesis of physical modeling via statistical mechanics, mathematical analysis via modern probabilistic techniques, and numerical computation via optimization algorithms. Rather than carry out dynamical simulations on fine grids, our approach is to derive governing equations for coarse-grained quantities and then to solve these equations computationally.

Accomplishments. We have developed a body of mathematical techniques that, when appropriately applied, settle many of the key questions arising in the statistical equilibrium theory of geophysical turbulence and related areas. Central among these techniques are a new set of large deviation estimates, which are powerful tools for calculating the asymptotic behavior of statistical mechanical models of a wide range of complex systems. In the context of Jupiter, our work is the first to apply a statistical equilibrium theory that both incorporates actual observational data from the Voyager and Galileo missions and predicts coherent structures consistent with those observations. Our new mathematical techniques are needed in several ways: 1) to select an appropriate statistical mechanical model that can be correlated with the observational data; 2) to link the statistical mechanical model with a maximum entropy principle that governs the coherent structures and is computationally tractable; 3) to prove the nonlinear stability of these coherent structures with respect to both microscopic fluctuations and macroscopic perturbations.

Significance. The emergence and stability of jetstreams on Jupiter and the coherent vortices that roll between them have been a genuine puzzle needing a theoretical explanation. Using a combination of powerful mathematical and computational techniques, we have succeeded in solving this puzzle by demonstrating that the observed features are the most probable states of a statistical mechanical model of the Jovian atmosphere. It must be emphasized that our work has significance well beyond the particular application to the Jovian atmosphere. For example, a similar theory explains the formation of solitons within dispersive wave turbulence governed by the nonlinear Schrödinger equation.

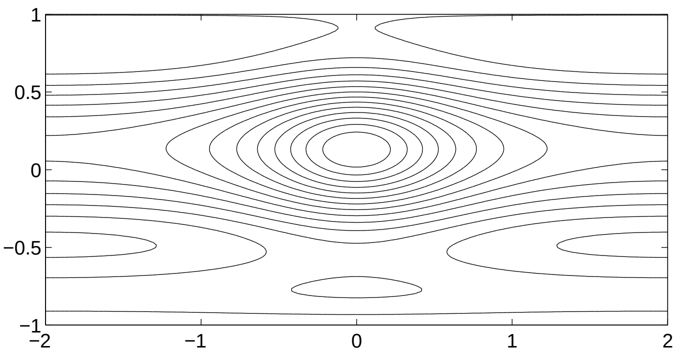

Figure 1: Mean streamline plot for an equilibrium state in a zonal band from $36.6^\circ S$ to $13.7^\circ S$ on Jupiter showing jetstreams and the Great Red Spot, which emerges at precisely the correct latitude.

Key Publications.

R. S. Ellis, K. Haven, and B. Turkington, "Large deviation principles and complete equivalence and nonequivalence results for pure and mixed ensembles." *J. Stat. Phys.* 101:999–1064 (2000).

R. S. Ellis, K. Haven, and B. Turkington, "Nonequivalent statistical equilibrium ensembles and refined stability theorems for most probable flows." *Nonlinearity*, in press (2001).

B. Turkington, A. Majda, K. Haven, and M. DiBattista, "Statistical equilibrium predictions of jets and spots on Jupiter." *Proc. Nat. Acad. Sci. USA* 98:12346–12350, 2001.

R. Jordan and B. Turkington, "Statistical equilibrium theories for the nonlinear Schrödinger equation." *Contemporary Math.* 283:27-39, 2001. *Proc. of AMS-IMS-SIAM Joint Summer Research Conference on Dispersive Wave Turbulence, June 2000.*

R. S. Ellis, R. Jordan, and B. Turkington, "A statistical approach to the asymptotic behavior of a class of nonlinear Schrödinger equations." In preparation (2001).