

THE FIRST AND SECOND FLUID APPROXIMATIONS TO THE LINEARIZED BOLTZMANN EQUATION (*)

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1. A. Introduction

Let $p_\varepsilon(t, x, \xi)$ be the solution of the Boltzmann equation :

$$(1.1) \quad \frac{\partial p}{\partial t} + \xi \cdot \text{grad } p = \frac{1}{\varepsilon} Q p,$$
$$\lim_{\varepsilon \downarrow 0} p(t, x, \xi) = f(x, \xi),$$

in a Euclidean domain D , where boundary conditions are prescribed on ∂D if D is finite. When $\varepsilon \rightarrow 0$, a great simplification occurs in the solution of (1.1), known as a "contraction of the description". This is formally treated by the Chapman-Enskog expansion at the physical level of rigor [17].

To make this precise, Grad [8] first considered (1.1) in a cube $D \subset \mathbb{R}^3$ with periodic boundary conditions, where Q is the linearized collision operator corresponding to a spherically symmetric potential function with a hard core. Using a priori estimates for (1.1), he proved that for f suitably smooth

$$(1.2) \quad T_\varepsilon(t)f - E(t)f = o(\varepsilon) \quad (\varepsilon \downarrow 0),$$

$$(1.3) \quad T_\varepsilon\left(\frac{t}{\varepsilon}\right)f - N_\varepsilon\left(\frac{t}{\varepsilon}\right)f = o(\varepsilon) \quad (\varepsilon \downarrow 0),$$

where $T_\varepsilon(t)f = p_\varepsilon$ is the solution of (1.1) and $E(t)$, $N_\varepsilon(t)$ denote, respectively, the solution operators for the linear Euler and Navier-Stokes equations with viscosity and heat conduction coefficients proportional to ε . These systems of partial differential equations are derived by means of the classical Chapman-Enskog-Hilbert expansion as applied to the linearized Boltzmann equation (1.1).

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