

## THE FIRST AND SECOND FLUID APPROXIMATIONS TO THE LINEARIZED BOLTZMANN EQUATION (\*)

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### 1. A. Introduction

Let  $p_\varepsilon(t, x, \xi)$  be the solution of the Boltzmann equation :

$$(1.1) \quad \frac{\partial p}{\partial t} + \xi \cdot \text{grad } p = \frac{1}{\varepsilon} Q p,$$
$$\lim_{\varepsilon \downarrow 0} p(t, x, \xi) = f(x, \xi),$$

in a Euclidean domain  $D$ , where boundary conditions are prescribed on  $\partial D$  if  $D$  is finite. When  $\varepsilon \rightarrow 0$ , a great simplification occurs in the solution of (1.1), known as a "contraction of the description". This is formally treated by the Chapman-Enskog expansion at the physical level of rigor [17].

To make this precise, Grad [8] first considered (1.1) in a cube  $D \subset \mathbb{R}^3$  with periodic boundary conditions, where  $Q$  is the linearized collision operator corresponding to a spherically symmetric potential function with a hard core. Using a priori estimates for (1.1), he proved that for  $f$  suitably smooth

$$(1.2) \quad T_\varepsilon(t)f - E(t)f = o(\varepsilon) \quad (\varepsilon \downarrow 0),$$

$$(1.3) \quad T_\varepsilon\left(\frac{t}{\varepsilon}\right)f - N_\varepsilon\left(\frac{t}{\varepsilon}\right)f = o(\varepsilon) \quad (\varepsilon \downarrow 0),$$

where  $T_\varepsilon(t)f = p_\varepsilon$  is the solution of (1.1) and  $E(t)$ ,  $N_\varepsilon(t)$  denote, respectively, the solution operators for the linear Euler and Navier-Stokes equations with viscosity and heat conduction coefficients proportional to  $\varepsilon$ . These systems of partial differential equations are derived by means of the classical Chapman-Enskog-Hilbert expansion as applied to the linearized Boltzmann equation (1.1).

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