THE LARGE DEVIATION PRINCIPLE FOR MEASURES WITH RANDOM WEIGHTS

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In this paper, we study the problem of large deviations for measures with random weights. We are motivated by previous work dealing with the special case occuring in the statistical mechanics of the Bose gas. We study the problem in an abstract setting, isolating what is general from what is dependent on Bose statistics. We succeed in proving the large deviation principle for a large class of measures with random weights and obtaining the corresponding rate function in an explicit form. In particular, our results are applicable to the Fermi gas and the spherical model.

1. Introduction

Measures with random weights arise naturally in statistical mechanics. By *measures with random weights* we mean measures of the form

$$m = \sum_j X_j \delta_{a_j} ,$$

where the a_j's are fixed points and the X_j's are random variables. These are to be contrasted with *empirical measures* where the opposite is the case, the X_j's being fixed numbers and the a_j's random variables. While there is a large literature on large deviation results for empirical measures (see, for example, [2] and [3]), not many people have addressed the problem of large deviations for measures with random weights, This problem has been studied mainly in the context of the Bose gas [4, 5]. For the Bose gas, the points a_j represent different momenta or energy levels, while the random variables X_j represent the number of particles at each a_j corresponding to Bose statistics. The present paper is motivated by the results of [4]. While we follow the general outline of [4], here we are interested in studying the problem in an abstract setting, isolating what is general from what is dependent on Bose statistics. We succeed in proving the large deviation principle for a large class of measures with random weights and obtaining the corresponding rate function in

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