

LARGE DEVIATIONS AND STATISTICAL MECHANICS

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ABSTRACT. This paper presents an overview of large deviations together with some applications to statistical mechanics. A general large deviation theorem is stated for sequences of random vectors for which a free energy function exists. This theorem is applied to three levels of large deviations for sequences of i.i.d. random vectors and to ferromagnetic models on \mathbb{Z}^D .

I. INTRODUCTION. In recent years, the theory of large deviations has undergone an extensive development. To a great measure, this development has been spurred by the important work of Donsker and Varadhan (1975a, 1975b, 1976, 1983). The roots of large deviation theory lie in a number of areas, including statistical mechanics, statistics, probability, and information theory. In statistical mechanics, the first important figure was Boltzmann, who in the 1870's studied the relation between entropy and probability in physical systems. Statistical mechanics also provides a natural context in which large deviation theory may be applied. This is shown, for example, by the lectures of Lanford (1973), who uses large deviation ideas to explain certain foundational questions in statistical mechanics. In my book [Ellis (1985)], I try to bridge the approach of Lanford and the work of Donsker and Varadhan. The present paper highlights some of the main topics in the book. I have not given detailed references here since the book gives them.

II. LARGE DEVIATION PROPERTY FOR RANDOM VECTORS. The theory of large deviations is concerned with the exponential decay of probabilities associated with certain stochastic systems. We first present a general framework for studying such problems. Afterwards, specific examples will be given. Let X be a complete separable metric space and $\{Q_n; n=1,2,\dots\}$ a sequence of probability measures on the Borel subsets of X . Suppose that as $n \rightarrow \infty$ $\{Q_n\}$ converges weakly to the unit point measure concentrated at some point x_0 in X ; i.e.,

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