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# Stochastic Networks

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# LARGE DEVIATION ANALYSIS OF QUEUEING SYSTEMS

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**Abstract.** In recent work we have succeeded in proving the large deviation principle for a general class of jump Markov processes that model queueing systems. The present paper highlights this result by giving the main ideas and omitting technicalities. Two key steps in the proof are to represent the large deviation probabilities as the minimal cost functions of associated stochastic optimal control problems and to study the limits of these probabilities using a subadditivity-type argument. This procedure leads to a characterization of the rate function that can be used either to evaluate it explicitly in the cases where this is possible or to compute it numerically in the cases where an explicit evaluation is not possible.

**1. Introduction.** The purpose of this paper is to highlight a new approach to the large deviation analysis of queueing systems which is developed in the paper [6]. Our aim here is to present the main ideas without giving all the technicalities needed for a rigorous treatment. Using this approach, not only do we prove the large deviation principle for a very wide class of such systems, but also derive an explicit formula for the rate function in a number of cases of interest, including tandem queues, open and closed Jackson networks, and certain two-dimensional stable queues. Such explicit formulas will be given in the paper [7]. Because of the generality of the model that we introduce, it should come as no surprise that an explicit formula for the rate function is in many other cases not available. Even when the rate function cannot be identified explicitly, our methods can also be used to calculate the rate function numerically. This aspect of our approach is currently under investigation.

To date the large deviation analysis of queueing systems has been restricted to those having special structures. In the case of tandem queues, contraction mapping techniques and the Skorohod problem have been used. These techniques originated in [1], were extended to domains with corners in [3,10], and have been applied to queueing systems in [13,16]. Using partial differential equation methods and in particular viscosity solutions, the paper [11] proves the large deviation principle for a class of open Jackson networks. A closed queueing model is treated in [12]. In addition, the results of Chapter 6 of [5] allow one to treat a general class of stable queueing systems in two dimensions that are modeled by discrete-time Markov chains [8]. Other one and two dimensional models and numerous applications are treated in the book [14]. In terms of generality, the results derived

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