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Multiple Phase Transitions in the Generalized Curie–Weiss Model

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The generalized Curie–Weiss model is an extension of the classical Curie–Weiss model in which the quadratic interaction function of the mean spin value is replaced by a more general interaction function. It is shown that the generalized Curie–Weiss model can have a sequence of phase transitions at different critical temperatures. Both first-order and second-order phase transitions can occur, and explicit criteria for the two types are given. Three examples of generalized Curie–Weiss models are worked out in detail, including one example with infinitely many phase transitions. A number of results are derived using large-deviation techniques.

KEY WORDS: Generalized Curie–Weiss model; specific Gibbs free energy; large deviations; first-order phase transition; second-order phase transition.

1. INTRODUCTION

The classical Curie–Weiss model is an exactly soluble model of ferromagnetism that allows one to study in detail the behavior of thermodynamic quantities in the neighborhood of the critical point. Unfortunately, the predictions of the model do not completely agree with experiment, and so other models, such as nearest neighbor Ising models, must be considered. However, because of its simplicity and because of the correctness of at least some of its predictions, the classical Curie–Weiss model occupies a central place in the statistical mechanics literature.

The classical Curie–Weiss model is a spin-1/2 model whose Hamiltonian is a quadratic function of the mean spin value in the system.

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