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FLUCTUATIONES IN CURIE-WEISS EXEMPLIS[1]

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1. Introduction

The primary topic of this paper is the statistics of mean field (or more accurately, Curie-Weiss) models. Although such models are often considered to possess trivial statistics (their fluctuations always being normally distributed), we shall see that when "properly" viewed, this is not the case. In fact, the probabilistic structure of these models is surprisingly rich and we present a detailed analysis of this structure with the hope that it will be helpful in analyzing analogous phenomena in less trivial models. Many of the results given here first appeared in [EN1] which can be consulted for detailed proofs. Sections 2-5 consist primarily of background material while sections 6-9 contain our main results.

2. General Ising Models

In this paper we will limit our attention to general Ising models (with pair interactions). For each $n = 1,2,\ldots$ we have a collection of (spin) random variables $\{X_i^n : i \in V_n\}$ where the V_n's are finite subsets of $\mathbb{Z}^d$ which tend to $\mathbb{Z}^d$ (in some appropriate sense) as $n \to \infty$; the joint distribution of $\{X_i^n\}$ is

$$(1) \qquad \frac{1}{Z_n} \exp\Big[\sum_{i,j \in V_n} J_{ij}(n) x_i x_j\Big] \prod_{i \in V_n} d\rho(x_i)$$

with ρ a finite measure on $\mathbb{R}^1$, and

$$(2) \qquad Z_n = \int \cdots \int \exp\Big(\sum J_{ij}(n) x_i x_j\Big) \prod d\rho(x_i)$$

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