

Chapman-Enskog-Hilbert Expansion for a Markovian Model of the Boltzmann Equation*

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Introduction

For a dilute gas of molecules of mass m subject to an external field \mathbf{F} , the equation governing the molecular distribution function $p = p(t, \mathbf{r}, \mathbf{v})$, $t \geq 0$, $\mathbf{r} \in \mathcal{R}^3$, $\mathbf{v} \in \mathcal{R}^3$, is the Boltzmann equation

$$(1) \quad \frac{\partial p}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} p + \frac{\mathbf{F}}{m} \cdot \frac{\partial}{\partial \mathbf{v}} p = B[p \otimes p].$$

The left-hand side of (1) is a streaming term which takes care of the local change in p due to the independent motion of the molecules. $B[p \otimes p]$ is a quadratic functional of p which depends on the intermolecular potential and which operates on p as a function of $\mathbf{v} \in \mathcal{R}^3$ only. It accounts for the change in p due to collisions between molecules.

The procedure by which one extracts the long time behavior of solutions of (1) and computes the macroscopic description of the gas in terms of the hydrodynamical quantities of pressure, temperature, etc., is known as the Chapman-Enskog development.¹ The basic assumption is that for times long compared to the mean time between collisions, the distribution function $p(t, \mathbf{r}, \mathbf{v})$ should have a *universal* functional form, so that the individual p depends in its details only upon the local hydrodynamical quantities:

$$(2) \quad \begin{aligned} \text{density } n(t, \mathbf{r}) &= \int d\mathbf{v} p(t, \mathbf{r}, \mathbf{v}), \\ \text{velocity } \mathbf{u}(t, \mathbf{r}) &= \int d\mathbf{v} \mathbf{v} p(t, \mathbf{r}, \mathbf{v}), \\ \text{temperature } T(t, \mathbf{r}) &= \frac{m}{3kn(t, \mathbf{r})} \int d\mathbf{v} (\mathbf{v} - \mathbf{u}(t, \mathbf{r}))^2 p(t, \mathbf{r}, \mathbf{v}), \end{aligned}$$

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¹ See Grad [6], pages 251-266; also Uhlenbeck and Ford [17], Chapter VI.

