Zentralblatt MATH Database 1931 – 2004

0566.60097

Ellis, Richard S.

Entropy, large deviations, and statistical mechanics. (English)

Grundlehren der Mathematischen Wissenschaften, 271. New York etc.: Springer-Verlag. XIV, 364 p. DM 184.00 (1985).

The book contains two parts. Part 1: Large deviations and statistical mechanics, consists of five chapters. Chapter 1: Introduction to large deviations, introduces levels-1,2 and 3 large deviations for independent identically distributed (i.i.d.) random variables with finite state space. Let S_n be the nth partial sum of the random variables. Let m_ρ be their common mean and P_ρ be the corresponding infinite product measure. Then the level-1 example is $P_\rho(|S_n/n - m_\rho| \geq \epsilon)$. Analogous probabilities for empirical measures and empirical processes give examples of level-2 and 3, respectively. In chapter 1 the exponential decay of large deviations probabilities in terms of entropy functions is obtained.

Chapter 2: Large deviation property and asymptotics of integrals, extends the ideas and results of the previous chapter by considering random vectors taking values in $\mathbb{R}^d$, $d \geq 1$. It is introduced a notion of exponential convergence which is a stochastic convergence, especially useful in statistical mechanics. The last section of this chapter presents Varadhan's theorem which gives the asymptotic behaviour of some class of integrals on a complete separable metric space. This allows to analyze the asymptotics of measures arising later in the book in the study of statistical mechanical models. Thus chapter 2 prepares the ground for applications of the theory of large deviations to equilibrium statistical mechanics in the next three chapters.

Chapter 3: Large deviations and the discrete ideal gas, is dedicated to the discrete ideal gas which in contrast to the standard ideal gas restricts the possible particle velocities to a finite subset of $\mathbb{R}$. This restriction is necessary for using the large deviation theory, developed in the previous chapter. It is proved that the microscopic sums converge exponentially to their assembly averages with respect to the microcanonical ensembles as the number of particles of the gas increases infinitely. The last section gives the definition of specific free energy and evaluates it by means of Varadhan's theorem in two different ways, based on the levels-1 and 3 large deviations, respectively. Evaluation by level-3 gives the Gibbs variational formula which expresses the specific free energy as a supremum over strictly stationary probability measures. This implies that for discrete ideal gas the level-3 equilibrium state, which is a strictly stationary probability measure on the infinite-particle configuration space, is a product measure.

Chapter 4: Ferromagnetic models on $\mathbb{Z}$, discusses the statistical mechanics of the Ising model of ferromagnetism, Curie-Weiss model and general ferromagnets on the one dimensional integer lattice $\mathbb{Z}$. The results of the chapter include properties of specific magnetization, specific Gibbs free energy, infinite-volume Gibbs states and phase transitions. Two approaches to studying the infinite-volume Gibbs states are developed. One based on the definition of infinite-volume Gibbs states as the closed convex hull of the set of weak limits of finite- volume Gibbs states. Another approach based on Gibbs variational principle, which characterizes the set of translation invariant infinite- volume

Gibbs states directly. The main results of chapter 4 include the following: exponential convergence properties of Gibbs states, FKG inequality, a large deviation proof of the Gibbs variational formula and the nonuniqueness of translation invariant infinite-volume Gibbs states with zero external field for the values of inverse temperature larger than the critical one when the latter is finite.

Chapter 5: Magnetic models on $\mathbb{Z}^D$ and on the circle, consists of two parts. Part 1 extends the results of chapter 4 to the case of ferro-magnetic models on the integer lattice $\mathbb{Z}^D$, $D \geq 1$, and derives the correlation inequalities. Part 2 presents a central limit theorem for total spin S_Λ in a symmetric hypercube Λ of $\mathbb{Z}^D$ in the case of uniqueness of infinite-volume Gibbs state. It is expected that at the critical point the central limit theorem breaks down. One of the sections discusses the question how to rescale the total spin S_Λ by a nonclassic scaling $|\Lambda|^\nu$, $\nu > 1/2$, such that $S_\Lambda/|\Lambda|^\nu$ has a nondegenerate limit in distribution (block spin scaling limit). At the end of this section an information about the form of this limit (e.g. Gaussian or non-Gaussian), which depends on the dimension of the lattice, is presented. The next section returns to the Curie-Weiss model extracting three faces. First, it shows that certain features of this model coincide with mean field theory. Second, it approximates general ferromagnetic models on $\mathbb{Z}^D$. Third, for the Curie-Weiss model it obtains that at the critical point $S_\Lambda/|\Lambda|^{34}$ converges in distribution to an explicitly determined, non-Gaussian random variable. Analyzing the circle model by large deviation techniques it is shown that the asymptotic behavior of the circle model depends strongly on the choice of interaction.

Part 2: Convexity and proofs of large deviation theorems, begins with the Chapter 6: Convex functions and the Legendre-Fenchel transform, discusses the properties of convex functions on $\mathbb{R}^d$ and the Legendre- Fenchel transform for these functions, referring for proofs to the book of *R. T. Rockafellar* [Convex analysis. (1970; Zbl 0193.184)]. The theory is applied in chapters 7-9 to derive the large deviation theorems and the properties of entropy functions which were stated without proofs in chapter 2.

Chapter 7: Large deviations for random vectors, is devoted to the proof of the main large deviation theorem which implies many of the large deviation results and applications. Also the properties of entropy functions are established.

Chapter 8: Level-2 large deviations for i.i.d. random vectors, gives an elementary self-contained proof of the level-2 large deviation property for i.i.d. random variables with a finite state space. The last two sections of chapter 8 give the proofs of the contraction principles relating level-1 and 2 for i.i.d. random vectors.

The last chapter 9: Level-3 large deviations for i.i.d. random vectors, presents the proof of the level-3 large deviation property for i.i.d. random variables with finite state space.

At the end of the book there are Appendices A-D. Appendix A: Probability, contains basic definitions and facts of probability theory which are needed in the book. It discusses the existence of probability measures on various product spaces and studies the properties of these measures (e.g. weak convergence, ergodicity). Appendix B: Proofs of two theorems in section 2.7, in particular, presents the proofs of Varadhan's theorem on the asymptotics of the integral stated in chapter 2. Appendix C: Equivalent notions of infinite-volume measures for spin systems, gives the definition of the infinite-volume Gibbs states for the case of many body interactions. It also discusses the equivalence between Gibbs states and other standard notions of infinite-volume measures. A sketch of the proof of the Gibbs variational formula and principle is given at the end of this

appendix. Appendix D: Existence of the specific Gibbs free energy, presents a proof of existence for free external conditions.
Each chapter of the book is followed by a notes section and by a problems section. There are over 100 problems, many of which have hints. The book may be recommended as a text, it provides a completely self-contained reading and is written in a rather detailed manner.

S.Pogosian

Keywords : statistical mechanics; empirical measures; empirical processes; specific free energy; Gibbs variational formula; Ising model of ferromagnetism; Curie-Weiss model; exponential convergence properties of Gibbs states; FKG inequality; ferro-magnetic models; central limit theorem; many body interactions

Classification :

∗60K35 Interacting random processes
82B05 Classical equilibrium statistical mechanics (general)
60F10 Large deviations
60-02 Research monographs (probability theory)

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