

## The GHS Inequality for a Large External Field

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We consider general even ferromagnetic systems with pair interactions in a nonnegative external magnetic field  $h$ . Classes of single-site measures  $\rho$  are found such that the GHS inequality is valid for all  $h \geq \tilde{h}$ , where  $\tilde{h} \geq 0$  is a number depending on  $\rho$  but independent of the size of the system. These measures include both absolutely continuous and discrete measures. For  $\rho \doteq a\delta_0 + \{(1-a)/2\} \cdot (\delta_1 + \delta_{-1})$ , some  $a \in [0, 1)$ ,  $\tilde{h}$  is determined exactly.

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**KEY WORDS:** GHS inequality; general even ferromagnetic systems; correlation inequalities.

### 1. INTRODUCTION

The Griffiths–Hurst–Sherman (GHS) inequality is a useful tool in the study of lattice spin systems with ferromagnetic pair interactions. For example, when valid, it implies that the average magnetization per site is a concave function of  $h \geq 0$ , where  $h$  denotes the external magnetic field. It also implies the absence of spontaneous magnetization except possibly at  $h = 0$ .<sup>(9)</sup> However, the validity of the GHS inequality for a particular system depends upon that system's single spin measures. For example, it holds for spin-1/2 systems<sup>(5)</sup>—i.e., for systems with single spin measures the Bernoulli measure  $\frac{1}{2}(\delta_1 + \delta_{-1})$ —but not for systems with single spin

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