Journal of Statistical Physics, Vol. 26, No. 1, 1981

The GHS Inequality for a Large External Field

Richard S. Ellis,[1,3] Charles M. Newman,[2,4] and Michael R. O'Connell[1]

Received November 5, 1980

We consider general even ferromagnetic systems with pair interactions in a nonnegative external magnetic field h. Classes of single-site measures ρ are found such that the GHS inequality is valid for all $h \geqslant \tilde{h}$, where $\tilde{h} \geqslant 0$ is a number depending on ρ but independent of the size of the system. These measures include both absolutely continuous and discrete measures. For $\rho = a\delta_0 + \{(1-a)/2\} \cdot (\delta_1 + \delta_{-1})$, some $a \in [0, 1)$, $\tilde{h}$ is determined exactly.

KEY WORDS: GHS inequality; general even ferromagnetic systems; correlation inequalities.

1. INTRODUCTION

The Griffiths–Hurst–Sherman (GHS) inequality is a useful tool in the study of lattice spin systems with ferromagnetic pair interactions. For example, when valid, it implies that the average magnetization per site is a concave function of $h \geqslant 0$, where h denotes the external magnetic field. It also implies the absence of spontaneous magnetization except possibly at $h = 0$.[9] However, the validity of the GHS inequality for a particular system depends upon that system's single spin measures. For example, it holds for spin-1/2 systems[5]—i.e., for systems with single spin measures the Bernoulli measure $\frac{1}{2}(\delta_1 + \delta_{-1})$—but not for systems with single spin

[1] Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01003.

[2] Department of Mathematics, University of Arizona, Tucson, Arizona 85721.

[3] Alfred P. Sloan Research Fellow. Research supported in part by National Science Foundation grant No. MCS 80-02149.

[4] Alfred P. Sloan Research Fellow. Research supported in part by National Science Foundation grant No. MCS 77-20683 and by the U.S.–Israel Binational Science Foundation.

0022-4715/81/0900-0037$03.00/0