Comments on Independent Events

This material is based on §3.4 of Ross's text. Let S be a sample space and P a probability.

Definition 1. (p 79). Events E and F are independent if $P(E \cap F) = P(E)P(F)$.

Prop 4.1 (p 81). If E and F are independent, then E and F^c are independent. Proof. Done in class (see p. 81).

Definition 2 (p 81). Three events E, F, and G are independent if $P(E\cap F\cap G) = P(E)P(F)P(G)$, $P(E\cap F) = P(E)P(F)$, $P(E\cap G) = P(E)P(G)$ and $P(F\cap G) = P(F)P(G)$.

Prop. 4.2 (not in book). If E, F, and G are independent, then E, F, and G^c are independent.

Proof. By Prop. 4.1 and Defn 2, the following are independent: E, F and E, G^c and F, G^c. We now prove $P(E\cap F\cap G^c) = P(E)P(F)P(G^c)$. To do this, note that $E\cap F$ and G are independent:

$P((E\cap F)\cap G) = P(E\cap F\cap G) = P(E)P(F)P(G) = P(E\cap F)P(G)$.

Thus by Prop 4.1 $E\cap F$ and G^c are independent. Since E, F indep.

$P(E)P(F)P(G^c) = P(E\cap F)P(G^c) = P(E\cap F\cap G^c)$. QED

Definition 3. Let $n \geq 2$ be an integer. Events $\{E_j, j=1,2,\dots,n\}$ are independent if $\forall k \in \mathbb{N}$, $2 \leq k \leq n$ and all subsequences of integers $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ $P\left(\bigcap_{\alpha=1}^{k} E_{i_\alpha}\right) = \prod_{\alpha=1}^{k} P(E_{i_\alpha})$.

Compare this with the definition in the text on p. 83, which defines events $\{E_j, j=1,2,\dots,n\}$ to be independent if for every subset $E_{1'}, E_{2'}, \dots, E_{r'}$, $r \leq n$

$$P(E_{1'} \cap E_{2'} \cap \cdots \cap E_{r'}) = P(E_{1'})P(E_{2'})\cdots P(E_{r'})$$

Which definition do you prefer?