The Probability of Winning at Craps Is 244/495 = 0.492929

Here are the rules of craps.

If you roll a total of 7 or 11 on the first roll, you win.

If you roll a total of 2, 3, or 12 on the first roll, you lose.

If you roll a total of 4, 5, 6, 8, 9, or 10 on your first roll, this number becomes your **point**. You continue to roll the dice. If you get your point total before a total of 7 appears, you win. If you roll a total of 7 before your point total appears, you lose.

Note: This is a 1:1 bet. That is, when you win, you win a dollar for each dollar that you bet.

The possible totals obtained from rolling two dice are shown at the right. Note that there are 36 cells containing the totals, and each cell has a probability of 1/36 of being the result of a craps roll.

	1	2	3	4	5	6
1	**2**	**3**	**4**	**5**	**6**	**7**
2	**3**	**4**	**5**	**6**	**7**	**8**
3	**4**	**5**	**6**	**7**	**8**	**9**
4	**5**	**6**	**7**	**8**	**9**	**10**
5	**6**	**7**	**8**	**9**	**10**	**11**
6	**7**	**8**	**9**	**10**	**11**	**12**

Notation. $(\alpha_j, \beta_j) = (\text{die 1, die 2})$ on j^{th} roll, $1 \le \alpha_j \le 6$, $1 \le \beta_j \le 6$.
For $2 \le k \le 12$ define $p_1(k)$ to be the probability of obtaining k in a roll of two dice. Thus

$$p_1(k) = P(\{\alpha_1, \beta_1\} : \alpha_1 + \beta_1 = k).$$

The table at the top of the page shows that

$$p_1(k) = 1/36 \text{ for } k = 2 \text{ or } 12,$$
$$p_1(k) = 2/36 \text{ for } k = 3 \text{ or } 11,$$
$$p_1(k) = 3/36 \text{ for k} = 4 \text{ or } 10,$$
$$p_1(k) = 4/36 \text{ for k} = 5 \text{ or } 9,$$
$$p_1(k) = 5/36 \text{ for k} = 6 \text{ or } 8,$$
$$p_1(k) = 6/36 \text{ for k} = 7.$$

On the next page we calculate the probability of winning.

Sample space $S^{(\infty)} = \{((\alpha_1,\beta_1),(\alpha_2,\beta_2),\dots) : 1 \le \alpha_j \le 6,\ 1 \le \beta_j \le 6,\ j \in \mathbb{N}\}$.

Denote the outcomes in $S^{(\infty)}$ by $x = ((\alpha_1,\beta_1),(\alpha_2,\beta_2),\dots)$.

Define $W = \{x \in S^{(\infty)} : x \text{ wins}\}$,

$W_7 = \{x \in S^{(\infty)} : \alpha_1 + \beta_1 = 7\}$, [7 on first roll]

$W_{11} = \{x \in S^{(\infty)} : \alpha_1 + \beta_1 = 11\}$. [11 on first roll]

For $k = 4, 5, 6, 8, 9, 10$, define

$W_k = \{x \in S^{(\infty)} : \alpha_1 + \beta_1 = k\}$, [k on first roll]

$W_{k,k}^{(2)} = \{x \in S^{(\infty)} : \alpha_1 + \beta_1 = k,\ \alpha_2 + \beta_2 = k\}$,
[point is k and obtain k on second roll]

and for $n \ge 3$ [point is k, obtain neither k nor 7 on rolls $2, 3, \dots, n-1$, obtain k on roll n]

$$W_{k,k}^{(n)} = \{x \in S^{(\infty)} : \alpha_1 + \beta_1 = k,\ \alpha_2 + \beta_2 \ne k \text{ or } 7,\ \dots,\ \alpha_{n-1} + \beta_{n-1} \ne k \text{ or } 7,\ \alpha_n + \beta_n = k\}$$

Then

$$W = W_7 \cup W_{11} \cup \bigcup_{4 \le k \le 6} \bigcup_{n=2}^{\infty} W_{k,k}^{(n)} \cup \bigcup_{8 \le k \le 10} \bigcup_{n=2}^{\infty} W_{k,k}^{(n)}$$

Since these events are mutually exclusive,

$$P(W) = P(W_7) + P(W_{11}) + \sum_{4 \le k \le 6} \sum_{n=2}^{\infty} P(W_{k,k}^{(n)}) + \sum_{8 \le k \le 10} \sum_{n=2}^{\infty} P(W_{k,k}^{(n)})$$

$$= P(W_7) + P(W_{11}) + 2 \sum_{4 \le k \le 6} \sum_{n=2}^{\infty} P(W_{k,k}^{(n)}).$$

The last step holds since $P(W_{4,4}^{(n)}) = P(W_{10,10}^{(n)})$, $P(W_{5,5}^{(n)}) = P(W_{9,9}^{(n)})$, and $P(W_{6,6}^{(n)}) = P(W_{8,8}^{(n)})$.