Problem #4, Homework 1: Hotel Coincidence Problem

This is related to the birthday problem. Suppose that family A spends 4 weeks during the year at a certain hotel. Find the minimum *n* such that if family B spends *n* weeks at the same hotel, then with probability greater than .5, family A and family B will spend the same week at the hotel. Here *n* is a positive integer less than or equal to 52. The weeks that each family spends at the hotel are not necessarily consecutive. As in the birthday problem, assume that the choice of weeks spent by family A and by family B at the hotel are random and independent.

Hints on Solving the Problem

Problem #4 in homework 1 is solved via a calculation that is somewhat analogous to that used to solve the birthday problem. In this email I give you several hints highlighting several features of this calculation that make it different from the calculation for the birthday problem.

Hint 1. Define Q_n to be the probability that in the 4 weeks that family A goes to the hotel family A does not meet family B during any of the n weeks that family B goes to the hotel. Show that the total number of weeks available to family A is the product C = 52·51· 50·49. How does this differ from the birthday problem?

Hint 2. In terms of n write down the total number of weeks W_n that in its 4 visits family A does not meet family B. For example, Q_1 equals the product 51·50·49·48.

Hint 3. It follows that Q_n equals the quotient W_n/C. Notice that Q_n is a decreasing function of n. By calculating Q_7, Q_8, and Q_9, show that n=9 is the smallest value of n such that Q_n < .5. Why is it okay to start with n =7?