Material To Review for Hour Exam #2

General Instructions

For each topic, review the material covered in class, homework problems in assignments #5–7, and related exercises. If a topic is listed and I do not indicate that you should memorize it, then understand the topic and proof (unless I indicate to omit the proof). When applicable, also review applications to coin tossing and dice rolling. All chapter, section, and page numbers refer to the 9th edition of the course text, *First Course in Probability* by Sheldon Ross.

Chapter 4. Random Variables

§4.1 Random Variables

a. **Memorize** definition of cumulative distribution function on page 116.

b. Examples

§4.2 Discrete random variables

a. **Memorize** definition of probability mass function on pages 116–117 and its properties.

b. **Memorize** definition of cumulative distribution function on page 118. Understand how to construct the cumulative distribution function from the probability mass function (see example on bottom half of page 118) and how to construct the probability mass function from the cumulative distribution function.

c. Examples

§4.3 Expected Value

a. **Memorize** definition on page 119.

b. Examples

§4.4 Expectation of a Function of a Random Variable

a. Proposition 4.1 on page 122 (omit proof)

b. Corollary 4.1 on page 125

c. Examples

§4.5 Variance

a. **Memorize** definition on page 126.

b. **Memorize** alternative formula for Var(X) on page 126: $E[X^2] – (E[X])^2$.

c. Examples

§4.6 The Bernoulli and Binomial Random Variables

a. **Memorize** definitions on page 127.

b. Examples

c. Let X_p be a Bernoulli random variable with parameter p. Memorize the derivation of the formulas E[X] = p and Var(X) = p(1-p).

d. **Memorize** relationship between binomial random variable $S_{(n, p)}$ with parameters (n, p) and sequence of independent Bernoulli random variables $X_{(i, p)}$ with parameter p: $S_{(n, p)} = \sum_{i=1}^n X_{(i, p)}$.

e. **Memorize** values of E[X] and Var(X) given on page 132, where $X = S_{(n, p)}$ (omit proofs)

§4.7 The Poisson Random Variable

a. **Memorize** definition on page 135

b. **Memorize** values of expected value and variance on pages 137–138 (omit proofs)

c. **Memorize** limit theorem relating binomial and Poisson random variables, which was discussed in class and can be obtained by sharpening the calculation on page 136. For details see handout #19 on course webpage. The statement of the limit theorem involves the following quantities.

- A Poisson random variable X_lambda with parameter lambda > 0.
- A binomial random variable S_(n, lambda/n) with parameters (n, lambda/n).

The limit theorem states that for i = 0, 1, 2, …, as n goes to infinity,

P{S_(n, lambda/n) = i} converges to P{X_lambda = i}.

d. **Memorize** Poisson approximation to the binomial distribution, which allows one to approximate P{S_(n, p) = i} by P{X_{np} = i}, where S_(n, p) is a binomial random variable with parameters (n ,p), X_{np} is a Poisson random variable with parameter n times p, and i is any nonnegative integer. The precise statement is that |P{S_(n, p) = i} - P{X_{np} = i}| \leq n p^2. See handout #19 for details.

e. Examples

Chapter 5. Continuous Random Variables

§5.1 Introduction

a. **Memorize** definition of continuous random variable and definition of probability density function on page 176

b. **Memorize** formula expressing density of a continuous random variable as derivative of cumulative distribution function on page 179

c. Examples

§5.2 Expectation and Variance of Continuous Random Variables

a. **Memorize** definition of E[X] on page 180

b. Proposition 2.1 on page 181 (omit proof)

c. Corollary 2.1 on page 183

d. **Memorize** definition of Var(X) on page 183

e. **Memorize** alternative formula for Var(X) on page 183

f. Examples

§5.3 The Uniform Random Variable

a. **Memorize** definition on page 184

b. Examples

§5.4 Normal Random Variables

a. **Memorize** definition on page 187 of probability density function of normal random variable N(\mu, \sigma^2). This pdf is written as f_{\mu, \sigma^2}(x).

b. **Memorize** values of E[X] and Var(X) on page 189, where X = N(\mu, \sigma^2)

c. **Memorize** the fact on page 189 that [N(\mu, \sigma^2) - \mu]/\sigma = N(0,1). See handout #21 for details.

d. Use of Table 5.1 on page 190

e. **Memorize** the formula on page 19) that for x > 0, \Phi(-x) = 1 - \Phi(x).

f. Examples

g, Handouts #22–25 are relevant for the topics in this section.

§5.4.1 The Normal Approximation to the Binomial Distribution

a. **Memorize** statement of the DeMoive-Laplace limit theorem on page 194

b. **Memorize** normal approximation to binomial distribution based on the DeMoivre-Laplace limit theorem, which is illustrated in the three examples in §5.4.1. Remember to use continuity correction.

c. Examples

§5.4 §5.4.1 Handouts #20–26 are relevant for the topics in these two sections..