Facts about $E[X]$ and $Var(X)$

These facts are valid in greater generality.

1) Let X and Y be discrete random variables and α and β real numbers. Then (p 148, 330)

$$E[\alpha X+\beta Y] = \alpha \cdot E[X] + \beta \cdot E[Y].$$

2) Let $X_1, X_2, \dots, X_n$ be discrete random variables and $\alpha_1, \alpha_2, \dots, \alpha_n$ real numbers. Then (p 148, 330)

$$E\left[\sum_{i=1}^{n} \alpha_i X_i\right] = \sum_{i=1}^{n} \alpha_i E[X_i].$$

3) Let X be a discrete random variable and α a real number. Then $Var(\alpha X) = \alpha^2 \cdot Var(X)$.

4) Discrete random variables X and Y are defined to be independent if for all x, y (p. 267)

$$P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$$

In this case, $Var(X+Y) = Var(X) + Var(Y)$ (p 357)

5) Let $X_1, X_2, \dots, X_n$ be discrete random variables that are pairwise independent. Then (p 357)

$$Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i).$$

6) Let X be a discrete random variable with $E[X]=\mu$ and $Var(X) = \sigma^2$. Then for any $\varepsilon > 0$ (p. 431)

$$P(|X-\mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}.$$ (Chebyshev's Inequality)

7) Let $X_1, X_2, \dots, X_n$ be discrete random variables that satisfy $E[X_i]=\mu$, $Var(X_i)=\sigma^2$ for each $i=1,2,\dots,n$ and are pairwise independent. Then for any $\varepsilon > 0$ (p 433)

$$P\left(\left|\frac{X_1+X_2+\cdots+X_n}{n} - \mu\right| \geq \varepsilon\right) \to 0 \text{ as } n \to \infty.$$

Proof. Combine 2), 5), and 6). This is the weak law of large numbers.