

MATH 331 – QUIZ 6  
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NAME:

The quiz consists of 4 problems. The maximum score is 10 points. The inverse Laplace transform of a function  $F(s)$  is denoted by  $\mathcal{L}^{-1}(F(s))$ .

1. (2.5 POINTS) Calculate the inverse Laplace transform  $\mathcal{L}^{-1}\left(\frac{s}{s^2+2}\right)$ .

$$\mathcal{L}^{-1}\left(\frac{s}{s^2+2}\right) = \mathcal{L}^{-1}\left(\frac{s}{s^2+\sqrt{2}^2}\right) = \cos(\sqrt{2}t)$$

2. (2.5 POINTS) Calculate the inverse Laplace transform  $\mathcal{L}^{-1}\left(\frac{1}{s^2+2}\right)$ .

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+2}\right) = \frac{1}{\sqrt{2}} \mathcal{L}^{-1}\left(\frac{\sqrt{2}}{s^2+\sqrt{2}^2}\right) = \frac{1}{\sqrt{2}} \sin(\sqrt{2}t)$$

3. (2.5 POINTS) Calculate the inverse Laplace transform  $\mathcal{L}^{-1}\left(\frac{1}{s^2+2s+3}\right)$ .

(Hint. Complete the square in the denominator.)

$$\begin{aligned}\mathcal{L}^{-1}\left(\frac{1}{s^2+2s+3}\right) &= \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2+\sqrt{2}^2}\right) \\ &= \frac{1}{\sqrt{2}} \mathcal{L}^{-1}\left(\frac{\sqrt{2}}{(s+1)^2+\sqrt{2}^2}\right) \\ &= \frac{1}{\sqrt{2}} e^{-t} \sin(\sqrt{2}t)\end{aligned}$$

4. (2.5 POINTS) Calculate the inverse Laplace transform  $\mathcal{L}^{-1}\left(\frac{s}{s^2 + 2s + 3}\right)$ .

(Hint. Complete the square in the denominator.)

$$\mathcal{L}^{-1}\left(\frac{s}{s^2 + 2s + 3}\right) = \mathcal{L}^{-1}\left(\frac{s}{(s+1)^2 + \sqrt{2}^2}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2 + \sqrt{2}^2}\right) - \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2 + \sqrt{2}^2}\right)$$

$$= e^{-t} \cos(\sqrt{2}t) - \frac{1}{\sqrt{2}} e^{-t} \sin(\sqrt{2}t).$$