MATH 331 – QUIZ 3
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October 21, 2016

NAME:

The quiz consists of 2 problems. The point totals for each problem are indicated. Please show all your work, make sure that the logic of your steps is clear, and indicate your answer clearly. The maximum score is 10 points. Do not simplify the arithmetic in your answers. The following notation is used: $y'(x) = dy(x)/dx$ and $y''(x) = d^2y(x)/dx^2$.

1. **2 POINTS**

Define $L[y](x) = y''(x) + ay'(x) + by(x)$, where a and b are constant coefficients. Parts (a) and (b) are each worth 1 point.

(a) For what values of the coefficients a and b does the homogeneous ODE $L[y] = 0$ have a fundamental set of solutions y_1, y_2 equal to e^x, xe^x?

(b) Assume that $L[y] = 0$ has a fundamental set of solutions y_1, y_2 equal to e^x, xe^x. Use the method of undetermined coefficients to determine your choice of the particular solution y_p of the nonhomogeneous ODE $L[y] = 6e^x$. Your answer should have the following form: $y_p(x)$ equals a constant C times a function of x. Do NOT evaluate the constant C.

a) A fundamental set e^x, xe^x means that the characteristic polynomial r^2+ar+b has a double root at 1, which means that the polynomial equals $(r-1)^2 = r^2-2r+1$. Thus $a=-2, b=1$, and $L[y] = y''-2y'+y$. $\boxed{\text{Answer: } a=-2,\ b=1.}$

b) $\boxed{\text{Answer: } y_p = Cx^2e^x}$ It is not necessary to explain your answer. Here is the explanation: because e^x, xe^x are a fundamental set, the original choice $y_p = Ce^x$ is changed to Cx^2e^x.

2. **8 POINTS**

Throughout this problem

$$L[y](x) = y''(x) - 5y'(x) + 4y(x).$$

Parts (a), (b), (c), and (d) are each worth 2 points.

(a) Find a fundamental set of solutions of the homogeneous ODE $L[y] = 0$.

(b) Use the method of undetermined coefficients to find a particular solution y_p of the nonhomogeneous ODE $L[y] = 6e^{2x}$.

(c) Determine the general solution of the nonhomogeneous ODE $L[y] = 6e^{2x}$.

(d) Solve the initial problem

$$L[y] = 6e^{2x},\ y(0) = 1,\ y'(0) = 4.$$

(a) $y'' - 5y' + 4y = 0$ corresponds to $r^2 - 5r + 4 = 0$
or $(r-4)(r-1) = 0$, which has roots 4 and 1. The
quadratic formula could also be used: $r = \frac{5}{2} \pm \frac{1}{2}\sqrt{9} = 4, 1.$
A fundamental set is $\boxed{e^{4x}, e^{x}}$ or $\boxed{e^{x}, e^{4x}}$

answer alternate answer

(b) Choose $\boxed{y_p = Ce^{2x}}$. This need not be modified
because e^{2x} does not solve $L[y] = 0$. Substituting gives
$(Ce^{2x})'' - 5(Ce^{2x})' + 4Ce^{2x} = 4Ce^{2x} - 10Ce^{2x} + 4Ce^{2x}$
$= -2Ce^{2x} = 6e^{2x}$, $-2C = 6$, $C = -3$, $\boxed{y_p = -3e^{2x}}$

(c) $y = C_1 e^{4x} + C_2 e^{x} - 3e^{2x}$ for fund. set e^{4x}, e^{x}
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(d) $y(0) = C_1 + C_2 - 3 = 1$ or $C_1 + C_2 = 4$ Subtracting:
$y'(0) = 4C_1 + C_2 - 6 = 4$ $\quad 4C_1 + C_2 = 10.$ $\quad 3C_1 = 6,\ C_1 = 2$
$C_2 = 4 - 2 = 2$

$\boxed{\text{Answer: } 2e^{4x} + 2e^{x} - 3e^{2x}}$

Here use $y = C_1 e^{4x} + C_2 e^{x} - 3e^{2x}$
Same answer w/ $y = C_1 e^{x} + C_2 e^{4x} - 3e^{2x}$