

MATH 331 – QUIZ 2  
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NAME:

The quiz consists of 3 problems. The point totals for each problem are indicated. Please show all your work, make sure that the logic of your steps is clear, and indicate your answer clearly. The maximum score is 10 points. Do not simplify the arithmetic in your answers. The following notation is used:  $y'(x) = dy(x)/dx$  and  $y''(x) = d^2y(x)/dx^2$ .

1. 4 POINTS

Solve the initial value problem  $y'(x) - y(x) = e^{6x}$ ,  $y(1) = 3$ .

The integrating factor is  $e^{-x}$ . Multiplying both sides of the ODE by  $e^{-x}$  gives  $(e^{-x}y)' = e^{-x}e^{6x} = e^{5x}$ . Integrating gives  $e^{-x}y = \frac{1}{5}e^{5x} + C$ , which we solve for  $y$ , obtaining

$$y(x) = \frac{1}{5}e^{6x} + Ce^x$$

Substituting  $y(1) = 3$ , we get  $3 = y(1) = \frac{1}{5}e^6 + Ce$ .

Thus  $C = e^{-1}(3 - \frac{1}{5}e^6)$ . The solution of the IVP is

$$y(x) = \frac{1}{5}e^{6x} + e^{-1}(3 - \frac{1}{5}e^6)e^x$$

2. 2 POINTS

What is the integrating factor for the ODE  $y'(x) + x^3y(x) = e^{x^{100}}$ ? Do NOT solve this ODE.

The integrating factor is  $\exp(\int x^3 + 3dt)$   
 $= e^{x^4/4}$

3. 4 POINTS

(a) (3 points) What is the general solution of the ODE  $y''(x) + 5y'(x) + 4y(x) = 0$ ?

(b) (1 point) Consider the initial value problem consisting of the ODE in part (a) together with the initial condition  $y(0) = 0$  and  $y'(0) = 0$ . What is the solution of this initial value problem?

(a) We substitute  $y = e^{rx}$ , obtaining  
 $r^2 + 5r + 4 = (r+4)(r+1) = 0$ . Thus  $r = -4, -1$ .  
General solution is  $y(x) = c_1 e^{-4x} + c_2 e^{-x}$ .

(b)  $0 = y(0) = c_1 + c_2$   
 $0 = y'(0) = -4c_1 - c_2$

Adding the 2 equations gives  $-3c_1 = 0$  or  $c_1 = 0$ .  
Also  $c_2 = -c_1 = 0$ . Thus the solution of the  
IVP is  $y(x) = 0 e^{-4x} + 0 e^{-x} = 0$  for all  $x$ .

Students can also write the solution  
 $y(x) = 0$  for all  $x$  without  
showing any work.