MATH 331 – QUIZ 1
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NAME:

The quiz consists of 3 problems. The point totals for each problem are indicated. Please indicate your answer clearly. The maximum score is 10 points. Do not simplify the arithmetic in your answers.

1. **4 POINTS**
   Solve the initial value problem $\dfrac{dy}{dx} = e^{\pi x},\ y(2) = 7.$

   Solution. Integrating both sides of the equation yields $y(x) = \frac{1}{\pi}e^{\pi x} + c$. Substituting the initial condition, we find that $7 = y(2) = \frac{1}{\pi}e^{2\pi} + c$. Thus $c = 7 - \frac{1}{\pi}e^{2\pi}$. The solution of the i.v.p. is
   $$y(t) = \frac{1}{\pi}e^{\pi x} + 7 - \frac{1}{\pi}e^{2\pi}.$$

2. **2 POINTS**
   Solve the initial value problem $\dfrac{dy}{dx} = e^{\pi x^3},\ y(2) = 7.$
   Hints: The integral of $e^{\pi x^3}$ cannot be calculated explicitly. Your answer should involve an appropriate integral.

   Solution. The integral of $e^{\pi x^3}$ cannot be calculated explicitly. Hence after integrating both sides of the equation, we obtain
   $$y(x) = \int^x e^{\pi t^3} dt \ + \ c.$$
   In order to evaluate c, we convert the indefinite integral to the definite integral $\int_2^x e^{\pi t^3} dt$. Substituting the initial condition, we find that $7 = y(2) = \int_2^2 e^{\pi s^3} ds + c = c$. Thus $c = 7$. The solution of the i.v.p. is
   $$y(x) = \int_2^x e^{\pi t^3} dt \ + \ 7.$$

3. **4 POINTS**
Determine an implicit relation satisfied by the solution of the initial value problem

$$\frac{dy}{dx} = \left(\frac{1}{2y + 6e^{6y}}\right) x^3,\ y(2) = 7.$$

Solution. This is a separable equation, which we rewrite as $(2y + 6e^{6y})dy/dx = x^3$. Integrating both sides of this equation with respect to the indicated variables yields the implicit relation $y^2 + e^{6y} = x^4/4 + c$. Substituting the initial condition, we find that $7^2 + e^{6\cdot 7} = 2^4/4 + c$. Thus $c = 49 + e^{42} - 4$. An implicit relation satisfied by the general solution of the i.v.p. is

$$y^2 + e^{6y} = x^4/4 + 49 + e^{42} - 4 = x^4/4 + 45 + e^{42}.$$