Table of Laplace Transforms

Table 6.1 Some Functions $f(t)$ and Their Laplace Transforms $\mathcal{L}(f) = \int_0^\infty e^{-st} f(t)\,dt$.

	$f(t)$	$\mathcal{L}(f)$	Values of s	Derivation
1	1	$1/s$	$s>0$	Example 1, p 205
2	t	$1/s^2$	$s>0$	Special case of formula 4
3	t^2	$2!/s^3$	$s>0$	Special case of formula 4
4	t^n $(n = 0, 1, \cdots)$	$\frac{n!}{s^{n+1}}$	$s>0$	p 207
OMIT 5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$		
6	e^{at}	$\frac{1}{s-a}$	$s>a$	Example 2, p 205
7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$s>0$	Example 4, pp 206-207
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$s>0$	Example 4, pp 206-207
9	$\cosh at$	$\frac{s}{s^2-a^2}$	$s>\lvert a\rvert$	Example 3, p 206
10	$\sinh at$	$\frac{a}{s^2-a^2}$	$s>\lvert a\rvert$	Example 3, p 206
11	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$	$s>a$	Formula 4 and Theorem 3, p 208
12	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$	$s>a$	Formula 4 and Theorem 2, p 208

Alternate derivation of formulas 7–8

By linearity of $\mathcal{L}$, $\mathcal{L}(e^{i\omega t}) = \mathcal{L}(\cos \omega t) + i\,\mathcal{L}(\sin \omega t)$.

As in the derivation of formula 6,

$$\mathcal{L}(e^{i\omega t}) = \frac{1}{s-i\omega} = \frac{s+i\omega}{s^2+\omega^2} = \frac{s}{s^2+\omega^2} + i\frac{\omega}{s^2+\omega^2} = \mathcal{L}(\cos \omega t) + i\,\mathcal{L}(\sin \omega t).$$

Thus $\mathcal{L}(\cos \omega t) = \frac{s}{s^2+\omega^2}$ and $\mathcal{L}(\sin \omega t) = \frac{\omega}{s^2+\omega^2}$.