Richard Ellis **F16_MATH_331_1**
Assignment Homework_Set_7 due 10/28/2016 at 11:00am EDT

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1. (1 pt) Suppose a spring with spring constant 8 N/m is horizontal and has one end attached to a wall and the other end attached to a 2 kg mass. Suppose that the friction of the mass with the floor (i.e., the damping constant) is 8 $\text{N}\cdot\text{s/m}$.

(1) Set up a differential equation that describes this system. Let x to denote the displacement, in meters, of the mass from its equilibrium position, and give your answer in terms of x, x', x''. Assume that positive displacement means the mass is farther from the wall than when the system is at equilibrium.

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(2) Find the general solution to your differential equation from the previous part. Use c_1 and c_2 to denote arbitrary constants. Use t for independent variable to represent the time elapsed in seconds. Enter c_1 as c1 and c_2 as c2.

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(3) Is this system under damped, over damped, or critically damped? [?] Enter a value for the damping constant that would make the system critically damped.
_____ $\text{N}\cdot\text{s/m}$

Correct Answers:

- `2*x''+8*x'+8*x = 0`
- `x = (c1 + c2 t) e^(-2 t)`
- `critically damped`
- `8`

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2. (1 pt) Suppose a spring with spring constant 8 N/m is horizontal and has one end attached to a wall and the other end attached to a mass. You want to use the spring to weigh items. You put the spring into motion and find the frequency to be 0.5 Hz (cycles per second). What is the mass? Assume there is no friction.

Mass = _________

Correct Answers:

- `0.810569 kg`

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3. (1 pt) Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 100\cos(8t)$ Newtons.

(1) Solve the initial value problem.

$y(t) =$

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(2) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$
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Correct Answers:

- `0.75 e^(-2t) cos(6t) + -1.08333 e^(-2t) sin(6t) + -0.7`
- `-0.75 cos(8t) + 1 sin(8t)`

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4. (1 pt) Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 40e^{-t}$ Newtons.

(a) Solve the initial value problem.

$y(t) =$

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(b) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$
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Correct Answers:

- `-0.540541 e^(-2t) cos(6t) + -0.0900902 e^(-2t) s`
- `0`

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5. (1 pt) Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton

(N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 30\sin(6t)$ Newtons.

(i) Solve the initial value problem.

$y(t) =$ ______

(ii) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$ ______

Correct Answers:

- `e^(-2t) (0.608108 cos(6t) + 0.101351 sin(6t)`
- `-0.608108 cos(6t) + 0.101351 sin(6t)`