

Only the Most Galvanizing Problems 2

On Wednesday April 26th, there will be a quiz in class containing one of these questions.

1. Wanting to get better at solving math problems, you decide to solve at least one math problem a day and at most 11 math problems per week for one year. Show that there must be a period of consecutive days in the year during which you will solve exactly 20 problems.
2. At some point during the quarter, your attendance record is less than 90%. Later in the quarter, your attendance record is more than 90%. Is there necessarily some time when your attendance record is exactly 90%? (Note that attendance does not vary continuously. If you have attended 3 out of 5 classes and then misses another one, then your attendance drops directly from 60% to 50%.)
3. Your apartment contains a long hallway with 10,000 light bulbs numbered from 1 through 10,000. Originally, all of the light bulbs are off.

On the first day, you pull the chain on every light bulb, turning them all on. On the second day, you pull the chain on every even-numbered light bulb, turning them off. On the third day, you pull the chain on every light bulb that's a multiple of three. This continues: on the n th day, you toggle all the light bulbs that are multiples of n . After 10,000 days, which light bulbs are on?

4. A positive integer is said to be *demonic* if it is written solely using 6's. For instance, 6, 66, 666 are all demonic. Prove that there exists a demonic number divisible by 2017.