1. Find the minimum size of a maximal matching in $C_n$. If $n$ is even, you can take every other edge.

2. Let $G$ be a bipartite graph. Show that $G$ has a matching of size at least $\frac{|E(G)|}{\Delta(G)}$.

3. Let $k$ be some fixed integer between 1 and $n$. Let $G$ be some subgraph of $K_{n,n}$ with more than $(k - 1)n$ edges. Prove that $G$ has a matching of size at least $k$.

4. Draw a connected, 3-regular graph that has both a cut vertex and a perfect matching.

5. Determine how many different perfect matchings there are in $K_{n,n}$. 


Hints:

1. Your answer should depend on $n$. How many edges in a row can you not have in a maximal matching?

2. Use König’s theorem. How many edges can a single vertex cover? So how many vertices must there be in any edge cover?

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4. Look at figure 1.106 in the book. Can you modify that figure to obtain the desired outcome?

5. Start building a matching. To how many vertices can you match the first vertex? The second?