Deceptively Uninspiring Homework 5
Due Wednesday May 21th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like \LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Given a positive integer $n$, a finite sequence of positive integers $(a_1, \ldots, a_k)$ is said to be coherent if $\sum_{i=1}^{k} a_i = n$. Show that the number of coherent sequences of $n$ is $2^{n-1}$. For instance, here are the $4 = 2^{3-1}$ coherent sequences of 3: (3), (2, 1), (1, 2) and (1, 1, 1).

2. Prove using induction that $n! > n^2$ for $n \geq 4$.

3. In a labeled tournament, the vertices are numbered from 1 to $n$. The labeling matters, but which vertex is drawn where doesn’t matter. Find (and prove) a formula for the number of labeled tournaments with $n$ vertices.

4. For $k \geq 3$, a $k$-cycle in a tournament is a sequence of vertices $v_1, v_2, \ldots, v_k$ with arcs $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_k \rightarrow v_1$. Prove that if a tournament contains a $k$-cycle for some $k > 3$, then it also contains a 3-cycle.