Deceptively Uninspiring Homework 5

Due Wednesday May 10th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like $\LaTeX$). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. (a) Show that $4x \equiv 3 (\mathrm{mod}\ 6)$ has no solutions with $0 \leq x < 6$.

   (b) Determine all solutions of $3x \equiv 7 (\mathrm{mod}\ 8)$ with $0 \leq x < 8$.

2. Let $f : X \to Y$ and $g : Y \to X$ be functions such that $f \circ g$ is the identity function I_Y on Y. This is to say that I_Y is the unique function with the property that $I_Y(y) = y$ for all $y \in Y$. Show that f is a surjection.

3. Is it possible for an equivalence relation to be a function? If so, under what conditions? If not, prove it.

4. Give an example of functions $f : A \to B$ and $g : B \to C$ such that f and g are **not** bijections, but $g \circ f$ **is** a bijection.

5. Let $f : \mathbb{R} \to \mathcal{P}(\mathbb{R})$ be the function defined by $f(x) = \{z \in \mathbb{R} : |z| \leq x\}$.

   (a) Is f injective?

   (b) Is f surjective?