Deceptively Uninspiring Homework 5
Due Wednesday May 10th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like \LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. (a) Show that $4x \equiv 3 \pmod{6}$ has no solutions with $0 \leq x < 6$.
   (b) Determine all solutions of $3x \equiv 7 \pmod{8}$ with $0 \leq x < 8$.

2. Let $f : X \rightarrow Y$ and $g : Y \rightarrow X$ be functions such that $f \circ g$ is the identity function $I_Y$ on $Y$. This is to say that $I_Y$ is the unique function with the property that $I_Y(y) = y$ for all $y \in Y$. Show that $f$ is a surjection.

3. Is it possible for an equivalence relation to be a function? If so, under what conditions? If not, prove it.

4. Give an example of functions $f : A \rightarrow B$ and $g : B \rightarrow C$ such that $f$ and $g$ are not bijections, but $g \circ f$ is a bijection.

5. Let $f : \mathbb{R} \rightarrow \mathcal{P}(\mathbb{R})$ be the function defined by $f(x) = \{z \in \mathbb{R} : |z| \leq x\}$.
   (a) Is $f$ injective?
   (b) Is $f$ surjective?