Deceptively Uninspiring Homework 2

Due Wednesday April 12th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Prove each of the following. For this exercise only, write your proofs in table form (like in Example 1.2 in the Conroy-Taggart) with a column for each **Step** and its **Justification**. Your Justifications may be any of the Axioms of the Integers or a previous part of this exercise.
   (a) If a and b are integers, then $(-a) \cdot b = -(ab)$.
   (b) If a and b are integers, then $(a+b)^2 = a^2 + 2ab + b^2$.
   (c) If $a + b = a$, then $b = 0$.
   (d) If a is an integer, then $a \cdot 0 = 0$.
2. Suppose a and b are integers. Prove each of the following.
   (a) If a is even and b is odd, then $a + b$ is odd.
   (b) If a and b are both odd, then $a + b$ is even.
   (c) If $a + b$ is odd, then a and b have opposite parity.
3. Suppose a and b are **negative** integers. Prove that, if $a < b$, then $a^2 > b^2$.
4. Suppose a and b are **positive** integers. Prove that, if $a \mid b$, then $a \leq b$.
5. Suppose $a > 0$ and $b \geq 0$ are integers such that $a \mid b$. Prove that, if $b < a$, then $b = 0$.