Deceptively Uninspiring Homework 1

Due Wednesday April 5th at the beginning of class

You may handwrite or type your answers/solutions/proofs. I highly encourage the use of a mathematical typesetting language (like LaTeX). If you handwrite, please make sure that your work is legible, and please staple your homework when you turn them in.

1. Determine whether each of the following is a statement. If it is, determine whether the statement is TRUE or FALSE. If the statement is TRUE, write a sentence or two explaining why. (This does not need to be a formal proof.) If the statement is FALSE, give a counterexample.
   (a) The sum of two odd numbers is even.
   (b) If a is an integer and $a \leq -5$, then $|a| > 5$.
   (c) Suppose a and b are integers. If $b > 0$, then $|a - b| < |a|$.
   (d) $x + y \geq x$
   (e) If a and b are integers such that $a + b$ is even, then a is even and b is even.
   (f) If a is a factor of b and a is a factor of c, then a is a factor of $b + c$.

2. Give a meaningful negation of each statement.
   (a) It is sunny and I am happy.
   (b) For every integer a, $|a| \geq 0$.
   (c) If a connected graph G has no odd cycles, then it is bipartite graph.
   (d) 3 is a factor of 17 or $7 < 10$.
   (e) If one uses riffle shuffles to shuffle a deck of cards, then 7 shuffles suffice. (If you're interested in the context, visit `http://www.nytimes.com/1990/01/09/science/in-shuffling-cards-7-is-winning-number.html`.)
   (f) There exists an integer a such that $a^{10} - 1 < 2$.
   (g) For all real x, if $x \neq 0$, then $x^2 > 0$.
   (h) For all integers a and b, if a and b are odd, then 4 is a factor of $a - b$ or 4 is a factor of $a + b$.
   (i) There is no such thing as a free lunch.
   (j) For every prime number p of the form $4k + 1$, there exist integers a and b such that $p = a^2 + b^2$.

3. Write the contrapositive of the following slogans.
   (a) "If it's not an iPhone, it's not an iPhone."
   (b) "If it isn't fresh, it isn't Legal."
   (c) "When you're here, you're family."

4. Write each of the following in *if-then* form and give its converse and contrapositive.
   (a) $n = 4k$ implies n is even.
   (b) Michelle says 'DUH' whenever Stephanie states the obvious.
   (c) $ab \geq 0$ whenever a and b are both negative.

5. Give, if possible, an example of a TRUE *if-then* statement for which:
   (a) the converse is true.
   (b) the converse is false.
   (c) the contrapositive is true.
   (d) the contrapositive is false.