Annie's Survival Kit 4 - Math 324

1. (10 points) Let $f(x,y) = x^2 - y^2 + 4xy$. Recall that $D_{\hat{\mathbf{u}}} f = \frac{df}{ds|_{\hat{\mathbf{u}}}} = \nabla f \cdot \hat{\mathbf{u}}$ and $\nabla f = \langle f_x, f_y \rangle$.
   (a) (3 points) In which direction does f **decrease** the fastest at $(2,1)$?
   (b) (1 point) For which unit vector does f **increase** the fastest at $(2,1)$?
   (c) (3 points) What is the rate of change of f at $(2,1)$ in the direction of the fastest **decrease**?
   (d) (3 points) Find all points at which the direction of fastest change of f is the same as in (a).

2. (10 points) Let $u = x^2 + y^2$, $v = \frac{y}{x}$ and $f = f(u,v)$.
   (a) (7 points) Express $xf_x + yf_y$ in terms of f_u, f_v, u and v.
   (b) (3 points) Find $xf_x + yf_y$ when $f(u,v) = u^3$.

3. (10 points) (a) (6 points) Find the tangent plane on $z = 2\sqrt{x^2+y^2}$ at the point $(1,-1,\sqrt{8})$.
   (b) (2 points) Is $(6,-8,-5)$ a normal vector for the tangent plane at some point of this surface? (Hint: the length and direction of the normal are irrelevant; only its orientation matters.) If so, find all such points on the surface. Otherwise, explain why.
   (c) (2 points) Is $(1,1,1)$ a normal vector for the tangent plane at some point of this surface? (Hint: the length and direction of the normal are irrelevant; only its orientation matters.) If so, find all such points on the surface. Otherwise, explain why.