Annie's Survival Kit 1 - Math 324

1. (10 points) Evaluate $\int_0^{\frac{1}{4}} \int_{\sqrt{y}}^{\frac{1}{2}} \frac{e^x}{x}\, dx\, dy$ by changing the order of integration.

   Hint 1: first figure out what is the integration region R.

   Hint 2: recall that $\int u\, dv = uv - \int v\, du$.

2. (10 points) (a) (5 points) Switch the order of integration of $\int_{-1}^{1} \int_{-\sqrt{2-x^2}}^{x} y\sqrt{x^2+y^2}\, dy\, dx$ to $dx\, dy$. Do not evaluate.

   (b) (5 points) Switch $\int_{-1}^{1} \int_{-\sqrt{2-x^2}}^{x} y\sqrt{x^2+y^2}\, dy\, dx$ to polar coordinates. Do not evaluate.

3. (10 points) (a) (7 points) Find the center of mass of a flat object with density δ proportional to the distance to the x-axis, and with region R bounded by $y = x^2 - 1$ and $y = 0$. (Recall that the center of mass $(\bar{x}, \bar{y})$ is such that $\bar{x} = \frac{\int\int_R x\delta\, dA}{\int\int_R \delta\, dA}$ and $\bar{y} = \frac{\int\int_R y\delta\, dA}{\int\int_R \delta\, dA}$.)

   (b) (3 points) Without doing further calculations, find the center of mass of a flat object with density proportional to the distance to the line $y = 3$, and with region R bounded by $y = x^2 + 2$ and $y = 3$.