Solutions for Practice Midterm 2

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Question 1

a) If we use the Laplace expansion formula for the last two rows we get

$$\det(B) = (-1)^{4+3} 6 \det(B_{43}) = (-6)(-1)^{3+3} 5 \det(A) = (-6)(5)(3) = -90$$

b) Since C is not invertible $\det(C) = 0$. Next we use the Laplace expansion for D for the last row

$$\det(D) = (1)\det(C) = 0.$$

Hence D is not invertible.

Question 2

a) The echelon form of A is

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -1 & -1 & -3 \\ 0 & 1 & 2 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Thus the pivot columns are:1,2,4,6 and the basis of $Col(A)$ is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

hence we can get a basis of $Nul(A)$:

$$\begin{bmatrix} -2 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ -2 \\ 1 \end{bmatrix}.$$

Question 3.

a) Since the columns of DC are the linear combinations of the columns of D, we conclude that that $Col(DC)$ is the linear subspace of $Col(D)$. On the other hand $\dim Col(D) = 2$ by assumption thus 3×3 matrix DC has rank at most 2 and is not invertible.
b) For example

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Problem 4.

a) No, not a basis. In the standard basis $S = \{t^2, t, 1\}$ the vectors in the problem have coordinates

$$\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0. \end{bmatrix}$$

A direct computation show that the echelon form of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ -3 & 3 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

has only two pivots.
b) the coordinates are the solution of the equation:

$$a_1(t^2 + t + 1) + a_2(t^2 + 2t + 1) + a_3 t = 3t^2 + t - 1.$$

The solution is the vector:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 10 \\ -4 \end{bmatrix}.$$

Problem 5.

a) No. For example $\det(3A) = 9\det(A)$, thus the transformation is not scaled correctly.
b) No. Multiplication by -1 does not preserve the unequality.
c) Yes. Sum of odd functions is odd and scalar of odd function is odd.

Problem 6.

The volume of the transform of the sphere is the volume of the original sphere times the absolute value of $\det(A)$. Since $\det(A) = 2$ we conclude that the volume is

$$(2)4\pi/3 = 8\pi/3$$

Also $\det(A^9) = \det(A)^9 = 2^9 = 512$.