MIDTERM 2 PRACTICE (425.1).

Problem 1. Find the linear orientation preserving mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ that send the parallelogram with vertices $(0, 0)$, $(2, 3)$, $(2, -1)$ $(4, 2)$ to the standard square with vertices $(0, 0), (1, 0), (0, 1), (1, 1)$.

Solution: The mapping is

$$T : (x, y) \mapsto (u, v), \quad u(x, y) = 3x/8 - y/4, \quad v(x, y) = x/8 + y/4.$$

Problem 2. Change the order of integration in the integral:

$$\int_0^1 \int_0^{1-x} \int_0^{x+y} f dz dy dx$$

to the orders

$$\iiint f dy dz dx, \quad \iiint f dx dy dz.$$

Solution:

$$\int_0^1 \left(\int_0^x \int_0^{1-x} f dy dz + \int_x^1 \int_{z-x}^{1-x} dy dz \right) dx =$$
$$\int_0^1 \left(\int_0^z \int_{z-y}^{1-y} f dx dy + \int_x^1 \int_0^{1-y} f dx dy \right) dz$$

Problem 3. Compute the integral

$$\iiint_V \sqrt{x^2 + y^2 + z^2} dx dy dz$$

where V is defined by $x^2 + y^2 + z^2 \leq z$.

Solution: $\pi/10$.

Problem 4. Compute the integral

$$\iint_V (x^2 + y^2) dx dy$$

where $x^4 + y^4 \leq 1$

Solution: $\ln(\sqrt{2\pi})$

Problem 5. Find the average of the function $e^{\sqrt{x^2+y^2+z^2}}$ over unit ball.

Solution: $3(e - 2)$.