# MIDTERM 1 PRACTICE (425.1).

**Problem 1.** Find the third order Taylor expansion of the function

$$f(x,y) = \sqrt{(1+x+y)}$$

at point $(x,y) = (0,0)$.

**Solution:**

$$1 + \frac{1}{2}(x+y) - \frac{1}{2}\frac{1}{2}\frac{1}{2}(x^2+2xy+y^2) + \frac{1}{6}\frac{1}{2}\frac{1}{2}\frac{3}{2}(x^3+3x^2y+3xy^2+y^3).$$

**Problem 2.** Find critical points of the function and determine its type (maximum, minimum or saddle point)

$$x^3 + y^3 - 3xy$$

**Solution:**

Point $(x,y) = (1,1)$ is a local minimum point.

**Problem 3.** Find critical points of the function and determine its type (maximum, minimum or saddle point)

$$e^{2x+3y}(8x^2 - 6xy + 3y^2)$$

**Solution:**

Local minimum at $(x,y) = (0,0)$; Saddle at $(x,y) = (-1/2, -1/4)$

**Problem 4.** Find constrained extremal point points of

$$f(x,y,z) = xyz, \text{ with } x^2+y^2+z^2 = 1, \quad x+y+z = 0.$$

**Solution:**

Minimal points are $\frac{1}{\sqrt{6}}(1,1,-2)$, $\frac{1}{\sqrt{6}}(-2,1,1)$, $\frac{1}{\sqrt{6}}(1,-2,1)$.

Maximum points are $\frac{1}{\sqrt{6}}(-1,-1,2)$, $\frac{1}{\sqrt{6}}(2,-1,-1)$, $\frac{1}{\sqrt{6}}(-1,2,-1)$.

**Problem 5.** Compute the integral

$$\int_R (x+y)dxdy$$

where $A$ is a square with vertices $(0,0), (1,0), (0,1), (1,1)$.

**Solution:** 1.

**Problem 6.** Write the integral

$$\int_R f(x,y)dxdy$$

as double integral if $R$ is a circle $x^2 + y^2 \leq y$.

**Solution:**

$$\int_{-1/2}^{1/2} \left( \int_{1/2-\sqrt{1/4-x^2}}^{1/2+\sqrt{1/4-x^2}} f(x,y)dy \right) dx = \int_0^1 \left( \int_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} f(x,y)dx \right) dy.$$

**Problem 7.** Change the order of integration in the integral:

$$\int_{-6}^{2} \left( \int_{x^2/4-1}^{2-x} f(x,y)dy \right) dx$$

**Solution:**

$$\int_{-1}^{0} \left( \int_{-2\sqrt{1+y}}^{2\sqrt{1+y}} f(x,y)dx \right) dy + \int_0^8 \left( \int_{-2\sqrt{1+y}}^{2-y} f(x,y)dx \right) dy.$$