

Ex

Find critical points of

$$\exp(1+x^2-y^2)$$

& use 2 der. test to classify them as local maxima or neither.

Soln

$$f_x = 2x \exp(1+x^2-y^2) \quad f_y = -2y \exp(1+x^2-y^2)$$

$$f_x \Rightarrow (0,0)$$

$$f_{xx} = 2(1+x^2) \exp(1+x^2-y^2)$$

$$f_{yy} = -2(1-y^2) \exp(1+x^2-y^2)$$

$$f_{xy} = -4xy \exp(1+x^2-y^2)$$

$$f_{xx} f_{yy} - f_{xy}^2 = (2e)(-2e) - 0 = -4e^2 < 0$$

Neither

Ex

Find global extr. values of $f(x,y) = 5x^2 - 2y^2 + 10$ on the disk $x^2 + y^2 \leq 1$.

First Soln

$$x = \text{boundary} = 5x^2 - 2y^2 + 10$$

$$x = \text{const} \quad y = \text{const}$$

$$f(t) = 5x^2 + 2y^2 - 2y^2 + 10$$

$$f'(t) = -14y + 2y = 0 \Rightarrow t = 0 \Rightarrow \text{II, II} \# 3, 2$$

$$(1,0) \quad (0,1) \quad (0,-1) \quad (0,0)$$

$$f(1,0) = 15 = f(-1,0) \quad f(0,1) = 8 = f(0,-1)$$

Insiders

$$f_x(x,y) = 10x$$

$$f_y(x,y) = -4y$$

Need test?

$$(0,0)$$

$$\Rightarrow \text{Max} = 15.$$