(4) Sec. der. test n vars.

$f: U \subset \mathbb{R}^n \to \mathbb{R}^n$ x_0 is crit pt.

$Hf(x_0)(h) > 0 \quad \forall h \neq 0 \Rightarrow$ max.

(5) Idea $f(x_0+h) = f(x_0) + Hf(x_0)(h) + \text{Rem}$, Rem $\leq \|h\|^3$.

(6) $n = 2$. $f: \mathbb{R}^2 \to \mathbb{R}$

(a) x_0 is min?

(i) $\frac{\partial f}{\partial x_j} = 0$

(ii) $\frac{\partial^2 f}{\partial x^2}(x_0, y_0) > 0$ (iii) $\det H > 0$. ← min

$<$ ← max.

< 0 saddle

(7) Global ext. $f: A \subset \mathbb{R}^n \to \mathbb{R}$ not need to be open.

(8) $D \subset \mathbb{R}^2$ closed $\Rightarrow$ f reaches max at some point of D. Abs. global max: $f(x) \leq f(x_0)$ $\forall x \in U$

(9) (i) crit pts $\subset U$
(ii) max of f on ∂U ← !
(iii) values from
(iv) concave values

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(1) Analogy with 1D. | (2) Boundaries / ∂D

1) Min. of $x^2 + 3y^2$ on $|(x, y)| < 1$