3.3

1) Def local min. of $f : U \subseteq \mathbb{R}^n \to \mathbb{R}$

is a pt s.t. $f(x_0) \le f(x)$ for $\forall x$ in some

nbhd of x_0

Resp. local max

2) First der. test

$U \subset \mathbb{R}^n$ $\quad f : U \in \mathbb{R}^n \to \mathbb{R}$

x_0 is crit. pt if $\frac{\partial f}{\partial x_1}(x_0) = 0 \quad \dots \quad \frac{\partial f}{\partial x_n}(x_0) = 0.$

Max, min $\Rightarrow$ crit.

Idea

1D 

we know

$g : \mathbb{R} \to \mathbb{R}$

$g(x_0)$ is max min $\Rightarrow g'(x_0) = 0.$

$g(t) = f(\vec{x}_0 + \vec{v}t) \quad \Rightarrow g'(t)\big|_{t=0} = 0 \Rightarrow$

$\nabla f(x_0) \cdot \vec{v} = 0 \quad \forall \vec{v} \quad \Rightarrow \nabla f(x_0) = 0.$

3) ~~Second test~~ Hessian

$f : U \subset \mathbb{R}^2 \to \mathbb{R}$ is C^2

$$Hf(x_0)(h) = \frac{1}{2} [h_1, h_2] \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$