PROBLEMS ON QUADRATIC FORMS AND LAGRANGE MULTIPLIERS

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The quadratic form is a homogeneous function of degree 2. That is function on $\mathbb{R}^n$:

$$Q(\vec{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_jx_j, \quad \vec{x} \in \mathbb{R}^n.$$

A square matrix A is symmetric if and only if $A_{ij} = A_{ji}$ for every i, j. Given a square matrix A we can construct quadratic form by:

$$Q_A(\vec{x}) = \vec{x}^t A\vec{x},$$

here we used t to denote transposition.

Suppose we have A, B symmetric square matrices of size $n \times n$. Let us also assume $\det(B) \neq 0$.

Problem 1 Set the Lagrange system for the constraint extremum problem for

$$Q_A(\vec{x}) \text{ restricted on the level set } Q_B(\vec{x}) = 1.$$

Show that that the corresponding Lagrange multipliers that solve the system are the eigenvalues of AB^{-1}. How do you interpret the critical values of the problem in terms of matrices A and B.

Problem 2. Suppose B is an identity matrix and $\det(A) \neq 0$. What can you say about the types of critical points from the previous problem if the signature of Q_B is

- $++$,
- $+-$?

What can you say about the types of critical points if the signature of Q_B is

- $++-$,
- $+++$?