

Quadratic forms

$$\sum_{i,j} a_{ij} x_i x_j = Q(x)$$

Let's n

Then there is a linear change of coord

$$y_i = \sum_{j=1}^n b_{ij} x_j \quad s.t.$$

$$Q(y) = \sum \lambda_i y_i^2$$

Algorithm

Complete the square

$$(1) \quad Q(x) = x_1^2 + 2x_1x_2 + 3x_2^2$$

$$= (x_1 + x_2)^2 - x_2^2 + 3x_2^2 = (x_1 + x_2)^2 + 2(x_2)^2$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

(2)

$$Q(x) = x_1^2 + 2x_1x_2 + 4x_1x_3 + x_2^2 + 5x_2x_3$$

$$= -x_3^2$$

$$= (x_1 + x_2 + 2x_3)^2 - x_2^2 - 4x_3^2 - 4x_2x_3 - x_2^2 + 5x_2x_3$$

$$= y_1^2 +$$

$$-5x_3^2 + x_2x_3$$

$$= x_2^2 - 2x_2x_3 + 5x_3^2 + 10x_2x_3 + 5x_3^2$$

$$\boxed{x_2 = z_2 + z_3, x_3 = z_2 - z_3}$$

$$y_1^2 =$$

$$-6z_2^2 + 10z_2z_3$$

$$-6z_3^2$$

$$y_2^2 =$$

$$-(2z_2 - 5z_3)^2 + 25z_3^2$$

$$= -y_2^2 + 19z_3^2$$

$$-6z_3^2$$