Math 235 Assignment 3 Due Thursday October 1, 2009

Part I: Webwork. Links for the Webwork part of the homework are on the course web page. Your user name is the part of your student e-mail address before the @ symbol, so if your e-mail address is red@student.umass.edu, then your username is red. Your password is your 8 digit student ID.

Part II: The first question is on a separate webpage.

2: We can represent the line through the point \( P \in \mathbb{R}^2 \) in the direction \( v \in \mathbb{R}^2 \) as the set of points
\[
\{ P + tv | t \in \mathbb{R}^2 \}.
\]

2a: Let \( P = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \). Sketch the points \( P + tv \) for \( t = -1, 0, 1, 1.5, 2 \).

2b: Let \( A \) be the matrix \( \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} \). Assume that \( Au \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \) if \( u \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \). Find a point and a direction so that \( A(L) \) is a line through that point in that direction.

**Notation:** If \( S \) is a set in \( \mathbb{R}^2 \), the \( A(S) \) is the set of points of the form \( A(x) \) for \( x \in S \). In our case \( L \) is the set we are interested in.

3: Let \( v = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \in \mathbb{R}^2 \). Consider the function
\[
F : \mathbb{R}^2 \rightarrow \mathbb{R}
\]
given by
\[
u \mapsto < v, u > .
\]

3a: Find a formula for \( F \).
3b: Show that \( F \) is linear. Hint: Write down the definition and use the properties of the dot product.
3c: Find the matrix that represents \( F \).

4: This question is considerably more "interesting" than the previous questions. I do not expect almost everybody to do all of this problem successfully.

4a: Let \( f \) be reflection about the \( x \)-axis; let \( g \) be reflection about a line \( L \) through the origin. Let \( h = g \circ f \). Then \( h \) is a reflection or a rotation. Which one? Answer this by making a series of convincing sketches that show how \( h \) acts on different points. Hint: Choose \( L \) to have an angle roughly equal to \( 15^\circ \).

4b: Let \( \theta \) be the angle that \( L \) makes with the \( x \)-axis. What are the matrices corresponding to \( f \) and \( g \). What is the matrix corresponding to \( h \)? Denote this matrix by \( H \). Is \( H \) a rotation matrix or is it a reflection matrix? If it is a rotation matrix, what is the angle of rotation in terms of \( \theta \)? If it is a reflection matrix, what is the line of reflection?