Rare Events, Information Theory, and Statistical Physics

A Conference Celebrating

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= 0.

that these probabilities te is given in terms of the rel

(Relative Entropy). Let $\rho = (\rho_1, \ldots, \rho_{\alpha})$ denote the proof. ve basic probabilistic model is defined. The relative

$$I_{\rho}(\gamma) = \sum_{k=1}^{\alpha} \gamma_k \log \frac{\gamma_k}{\rho_k}.$$

lative entropy are given in the next len volving relative entropy [e.g., Prop. 4. than calculus to determine where I convexity inequality is that for $x \ge 1$

neasures the discrepancy between ly if $\gamma=
ho$. Thus $I_{
ho}(\gamma)$ attains its i n, $I_{
ho}$ is strictly convex on \mathcal{P}_{lpha} ; that

$$+(1-\lambda)\nu) < \lambda I_{\rho}(\mu) + (1-\lambda)I(\nu)$$

of the strictly convex function $x \log x$ ha $\geq x-1$ with equality if and only if x=

$$\frac{\gamma_k}{\rho_k} \log \frac{\gamma_k}{\rho_k} \ge \frac{\gamma_k}{\rho_k} - 1$$

by if and only if $\gamma_k = \rho_k$. Multiplying this inequality

$$I_{\rho}(\gamma) = \sum_{k=1}^{\alpha} \gamma_k \log \frac{\gamma_k}{\rho_k} \ge \sum_{k=1}^{\alpha} (\gamma_k - \rho_k) =$$

0 if and only if $\gamma = \rho$. If $\gamma = 1$

of Large Deviations

r words, we want

[z-a,z]

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Richard S. Ellis

o the object under study in the present section. For φ $L_n(y) = L_n(\omega, y) = \frac{1}{n} \sum_{i=1}^n \delta_{X_j(\omega)}$

 X_j form a sequence of i.i.d. random variables with comp

unts the relative frequency with which $(j) = n^{-1} \cdot \operatorname{card}\{j \in \{1, \dots, n\} : \omega_j\}$

ın?

Boltzmann

. For the purpo

of a probabilistic qua

ction. Thus let $\alpha \geq 2$ be

 α a set of α positive real number

measure on $\Omega_n = \Lambda^n$ with one dimension

 $(\omega_n) \in \Omega_n$, we let $\{X_j, j=1,\ldots,n\}$ be the co

$$L_n = L_n(\omega) = (L_n(\omega, y_1))$$
$$= \frac{1}{n} \sum_{j=1}^n (\delta_{X_j(\omega)} \{y_1\})$$

mple mean of the i.i.d. random ve of probability vectors

$$\mathcal{P}_{\alpha} = \left\{ \gamma = (\gamma_1, \gamma_2, \dots, \gamma_{\alpha}) \in \mathbb{R}^{\alpha} \right\}$$

behavior of L_n is straightforward to de r any $\gamma \in \mathcal{P}_{\alpha}$ and $\varepsilon > 0$, we define the op

$$B(\gamma, \varepsilon) = \{ \nu \in \mathcal{P}_{\alpha} : \|\gamma - \nu\| \}$$

ve the common distribution ρ , for each $y_k \in \Lambda$

$$P_n\{L_n(y_k)\} = E^{P_n}\left\{\frac{1}{n}\sum_{j=1}^n \delta_{X_j}\{y_k\}\right\} = \frac{1}{n}\sum_{j=1}^n P_n\{X_j\}$$

notes expectation with respect to \underline{P} means of i.i.d. random variabl

on the Theory of





Speakers

Paul Dupuis (Brown University) Jon Machta (UMass Amherst)

Charles Newman (Courant Institute, NYU)

Peter Otto (Willamette University)

Hugo Touchette (Stellenbosch University, South Africa) Bruce Turkington (UMass Amherst)

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