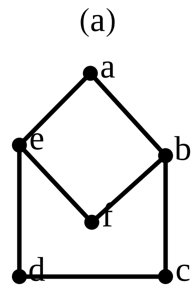
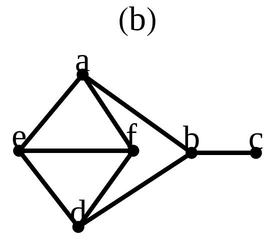
Math 455.1 **Homework Set 9** **Spring, 2009**

Due: Monday, May 11

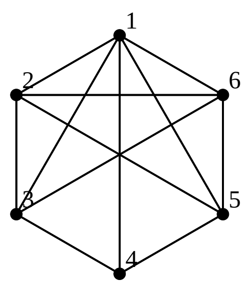
Instructions: Work either individually or in a team.

1. Use the greedy vertex-coloring algorithm to color the vertices of the following graphs. In each case, tell whether the result is optimal.





2. (a) Without actually finding any vertex coloring yet, use the theory we've developed in order to obtain a lower and an upper bound on the chromatic number of the graph:

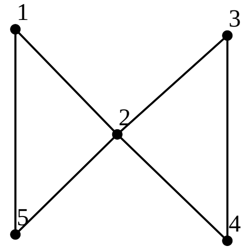


   (b) Now determine the actual value of $\chi(G)$.

3. Finish the solution of our procedure-scheduling problem: Find a minimal vertex coloring of the associated graph. Then interpret the results in terms of scheduling the procedures. (See the handout "Procedure scheduling problem", 27 April 2009.)

4. Use induction to prove that every tree with exactly n vertices has exactly $n-1$ edges. (*Note.* Be careful: the proposition you want to prove about n is that *every* tree with exactly n vertices has exactly $n-1$ edges.)

5. (a) Find *all* spanning trees of the following graph. (The order in which you add vertices or edges to get a particular spanning tree is irrelevant; all that matters is what the particular spanning tree is.)



(b) Use the depth-first search algorithm, beginning at vertex 1, to find a spanning tree for the following graph: