Math 455.1 **Homework Set 7** **Spring, 2009**

Due: Monday, April 27

Instructions: Work either individually or in a team.

1. (a) With paper and pencil/pen draw a representation of the complete bipartite graph $K_{2,3}$ and tell how many edges it has. Then do the same for $K_{3,3}$

   (b) In general, how many edges does the complete bipartite graph $K_{m,n}$ have? Why?

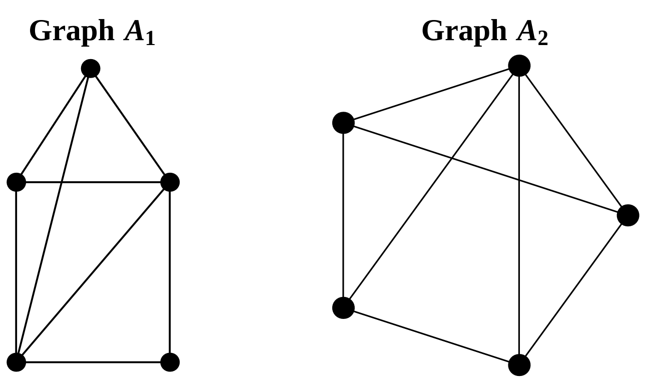
2. Use the *Combinatorica* package to create and display graphs that are isomorphic to the complete bipartite graph $K_{3,3}$ by each of the following methods. By means of suitable options from the package, number or label the vertices in each display so as to indicate isomorphisms of all the displayed graphs—that is, vertices that correspond under an isomorphism should be labeled the same.

   - by means of `CompleteGraph`;
   - by forming the **join** of two graphs that are isomorphic to each other but neither of which is isomorphic to $K_{3,3}$;
   - by using `FromOrderedPairs` to build the graph.

3. For each of $k = 2, 3, 4, 5$, determine the number of **walks** of length exactly k between two *adjacent* vertices of $K_{3,3}$. Reason directly about the structure of the graph, and indicate your reasoning! (You may use a graph drawing from Problem 1 or 2, above, to guide your reasoning. But you should not arrive at your answers merely by enumerating all the walks of the specified lengths.)

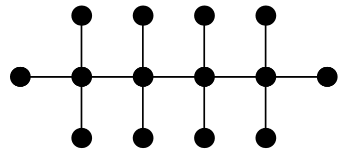
4. In each part, determine whether the two graphs are isomorphic. If they are, construct a specific isomorphism; if they are not, prove that they are not by finding a graph property that one has but the other does not have.

   (a) The graphs A_1 and A_2 shown below.

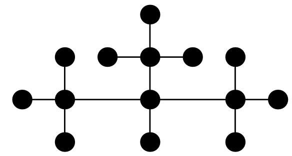


   (b) The graphs B_1 and B_2 shown below.

Graph B_1



Graph B_2



5. (a) Draw—paper and pencil will suffice—a connected graph that has exactly 5 vertices and exactly $5 - 1 = 4$ edges.

   (b) Prove that a connected graph with n vertices has at least $n-1$ edges. [*Suggestion:* Use induction on n.]

6. (a) Use a suitable *Combinatorica* function to construct an Eulerian trail in each of the complete graphs K_5 and K_7. Display these Eulerian trails by using suitable options from *Combinatorica* to number the successive *edges* 1, 2, 3, etc., in the order they are traveled in that Eulerian trail.

   (b) Prove **Poinsot's Theorem**: The complete graph K_n is Eulerian if and only if n is odd. (*Note:* This is a two-way implication!)