Due: Monday, March 30—correction! (start of class)

- Again work individually, that is, not in a team. See the About > Homework sets page on the Math 455 web site regarding collaboration and plagiarism.

For Mathematica work here, turn in printed pages. Try to place associated written work directly onto such printed pages.

1. Use Mathematica to do Exercise 6 from notebook Divisibility.nb. Turn in for your solution a printout of just the relevant input-output cell pairs along with a text cell containing your answer to the question in (b).

2. (a) Use long division to find \( 99666 \div 198 \) and \( 99666 \mod 198 \).
   
   (b) Use Mathematica to find \( 99666 \div 198 \) and \( 99666 \mod 198 \).

3. An integer \( n \) is said to be even when it is divisible by 2, in other words, if \( n = 2k \) for some integer \( k \). And \( n \) is said to be odd when it is not even. Prove that \( n \) is odd if and only if \( n = 2k + 1 \) for some integer \( k \).

4. (a) Find the prime factorizations of 99666 and 198 by directly checking the primes not exceeding the square-root of each.
   
   (b) Find the prime factorizations of 99666 and 198 by using a built-in Mathematica function, and type (or write by hand) what these prime factorizations are. (Hint: Look at Divisibility.nb. Or search the Documentation Center for “prime factor”.)
   
   (c) Use the prime factorizations of 99666 and 198 to find their greatest common divisor.

5. (a) Use the Euclidean algorithm to find \( \gcd(99666, 198) \). Show the steps!
   
   (b) Use your work in (a) to express \( \gcd(99666, 198) \) as an integer linear combination of 99666 and 198. Show the steps!

6. (a) Define a Mathematica function \( \text{gcd recursively} \) that implements the Euclidean algorithm to find the greatest common divisor of arbitrary nonnegative integers (not both 0). For example, \( \gcd[24, 36] \) should have result 12.
   
   Your definition should consist of just a couple of simple Mathematica expressions; it should not use If, Which, or Switch. Since the definition should be recursive, not iterative, it should not use While, Do, or For. (Of course, do not use the built-in function GCD.)
   
   (b) Use your function \( \text{gcd} \) to find \( \gcd(99666, 198) \) again. Verify the result by using the built-in function GCD, too.

7. Suppose \( a \) and \( b \) are relatively prime positive integers both of which divide the positive integer \( n \). Deduce that the product \( ab \) must divide \( n \), too.

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\(^1\)See Factorial.nb and Fibonacci.nb for examples of recursively defined functions of one variable and two variables, respectively.