Math 455.1 **Homework Set 4** **Spring, 2009**

Due: Monday, March 30—*correction!* (start of class)

- Again work ***individually***, that is, *not* in a team. See the About > Homework sets page on the Math 455 web site regarding collaboration and plagiarism.

  For *Mathematica* work here, turn in printed pages. Try to place associated written work directly onto such printed pages.

1. Use *Mathematica* to do Exercise 6 from notebook `Divisibility.nb`. Turn in for your solution a printout of just the relevant input-output cell pairs along with a text cell containing your answer to the question in (b).

2. (a) Use long division to find 99666 div 198 and 99666 mod 198.

   (b) Use *Mathematica* to find 99666 div 198 and 99666 mod 198.

3. An integer n is said to be **even** when it is divisible by 2, in other words, if $n = 2\,k$ for some integer k. And n is said to be **odd** when it is not even. Prove that n is odd if and only if $n = 2\,k + 1$ for some integer k.

4. (a) Find the prime factorizations of 99666 and 198 by directly checking the primes not exceeding the square-root of each.

   (b) Find the prime factorizations of 99666 and 198 by using a built-in *Mathematica* function, and type (or write by hand) what these prime factorizations are. (*Hint:* Look at `Divisibility.nb`. Or search the Documentation Center for "prime factor".)

   (c) Use the prime factorizations of 99666 and 198 to find their greatest common divisor.

5. (a) Use the Euclidean algorithm to find $\gcd(99666, 198)$. Show the steps!

   (b) Use your work in (a) to express $\gcd(99666, 198)$ as an integer linear combination of 99666 and 198. Show the steps!

6. (a) Define a *Mathematica* function `gcd` **recursively**[1] that implements the Euclidean algorithm to find the greatest common divisor of arbitrary nonnegative integers (not both 0). For example, `gcd[24,36]` should have result `12`.

   Your definition should consist of just a couple of simple *Mathematica* expressions; it should *not* use `If`, `Which`, or `Switch`. Since the definition should be recursive, not iterative, it should not use `While`, `Do`, or `For`. (Of course, do *not* use the built-in function `GCD`.)

   (b) Use your function `gcd` to find $\gcd(99666, 198)$ again. Verify the result by using the built-in function `GCD`, too.

7. Suppose a and b are relatively prime positive integers both of which divide the positive integer n. Deduce that the product $a\,b$ must divide n, too.

[1] See `Factorial.nb` and `Fibonacci.nb` for examples of recursively defined functions of one variable and two variables, respectively.