Math 455.1 **Homework Set 3** **Spring, 2009**

Due: Wednesday, March 4 (start of class)

- For this homework set, you should work ***individually***, that is, *not* in a team. See the About > Homework sets page on the Math 455 web site regarding collaboration and plagiarism.

  For *Mathematica* work here, turn in printed pages—just what's directly relevant. You may, and in fact are encouraged, to place associated written work directly onto such printed pages (provided it's neat and easy to find).

1. When we form the **binary (base 2) representation**

$$k = (b_{n-1} \cdots b_3 b_2 b_1 b_0.)_2$$

of a positive integer k, we mean that each "bit" $b_j \in \{0, 1\}$ for $j = 0, 1, 2, \ldots, n-1$ and

$$k = \sum_{j=0}^{n-1} b_j \, 2^j .$$

Such a binary representation form uses n bits. For example,

$$23 = (1\ 0\ 1\ 1\ 1)_2 \qquad \text{and} \qquad 23 = (0\ 1\ 0\ 1\ 1\ 1)_2$$

are binary representations of 23 using 5 bits and 6 bits, respectively. (Of course, the leading bit, 0, in the latter representation is superfluous.)

   (a) Use an appropriate formula from the text or lecture to determine the largest positive integer that has a binary representation using n bits.

   (b) Use your formula in *Mathematica* to make a table with two columns, the first column showing the number n of bits for $n = 1, 2, \ldots, 10$ and the second column giving that largest n-bit positive integer.

2. (a) Do a suitable experiment in *Mathematica* to find the first K such that $n! > 3^n$ seems to be true for all $n \geq K$.

   (b) Use mathematical induction to prove conclusively the result you found experimentally in (a).

3. Do Review Exercise 2.5.2 on page 41 of the text. Specifically:

   (a) Do the experimentation with *Mathematica*...

   - first, form a 2-column table in which the first column shows values of n for $n = 1, 2, \ldots 12$ and the second column shows the corresponding sum from 2.5.2 as calculated by *Mathematica* (*do* not *turn in this input or output*);
   - second, state your conjecture of a "closed-form formula", in terms of n, for that sum; and
   - experimentally confirm your conjecture by forming a 3-column table in which the first column shows values of n for $n = 1, 2, \ldots 12$, the second column again shows the corresponding sum as calculated by *Mathematica*, and the third column shows the corresponding values calculated from your closed-form formula. (*Do* turn in the printed input and output for this table.)

(Of course, the 2nd and 3rd columns should turn out to be the same if your conjectured formula is correct.)

(b) Use mathematical induction to prove that your formula is correct.

(c) *Extra credit:* Use a combinatorial argument to prove the same formula.

4. (a) A questionnaire about campus life is sent to 13 sophomores, 15 juniors, and 20 seniors. How many questionnaires, in all, must be returned in order that at least 9 are received from members of the same class-year?

   (b) Same as (a), except now the questionnaire is also sent to 5 freshmen.

5. (a) In how many ways can the letters of `RECURRENCERELATION` be arranged?

   (b) In how many ways can the letters of `RECURRENCERELATION` be arranged so that no two of the `E`s are next to one another?

   (c) In how many ways can the letters of `RECURRENCERELATION` be arranged so that at least two of the `E`s are next to one another?