Math 455.1 **Homework Set 2** **Spring, 2009**

Problems from textbook quoted in their entirety here

Due: Monday, February 23 (start of class)

- For this homework set, you should again work in a team of 2 or 3 and turn in a *single* paper for the entire team. It might be a good idea to form different teams than for Set 1. See further details on the About→Homework Sets page of the course web site.

  For *Mathematica* work here, turn in printed pages—just what's directly relevant. You may, and in fact are encouraged, to place associated written work directly onto such printed pages (provided it's neat and easy to find).

- See further instructions about homework **format** on the web site.

1. The text uses algebraic manipulations to arrive at the **Addition Formula**:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \qquad (k = 1, 2, \ldots, n-1, \quad n = 2, 3, \ldots)$$

   Give another justification of this formula, by interpreting the three binomial coefficients involved in terms of number of subsets.

2. Review Exercise 1.8.21 on page 23: Let B be a subset of A, $|A| = n, |B| = k$. What is the number of all subsets of A whose intersection with B has 1 element?"

In the three problems below, arrive at the final answer:

(i) first, by using principles taught in this course, *not* brute-force enumeration of all possibilities, stating. . .

- which combinatorics principle or principles are being used [for example, $\#(A \times B) = \#(A)\#(B)$; the Principle of Inclusion-Exclusion; etc];
- what the relevant mathematical model is in terms of sets or functions (for example, a product of sets; the set of all functions from one set to another; etc.); and
- what the constituent sets are that go into the model (for example, which finite sets you are forming the product of; the sets constituting the domain and codomain for the set of functions). . .

(and you may use *Mathematica* to do the arithmetic to get an actual numeric answer); and then, as a check,

(ii) by forming the set of all possibilities through use of relevant functions from notebooks `SetsAndFunctions.nb` or `Combinatorics.nb`—do *not* print the lists constituting these sets, please!—and then using `Card` to count these possibilities.

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3. Review Exercise 2.5.5 on page 41 of the text: "There is a class of 40 girls. There are 18 girls who like to play chess, and 23 who like to play soccer. Several of them like biking. The number of those who like to play both chess and soccer is 9. There are 7 girls who like chess and biking, and 12 who like soccer and biking. There are 4 girls who like all three activities. In addition we know that everybody likes at least one of these activities. How many girls like biking?"

4. A Unix password can consist of 6 to 8 characters. Each character can be alpha-numeric (that is, an upper-case letter, a lower-case letter, or a digit `0`, `1`, `2`, ..., `9`) or one of the twelve non-alphanumeric characters `!`, `#`, `$`, `=`, `@`, `^`, `&`, `*`, `_`, `+`, `(`, `)`. But a password must include at least one non-alphanumeric character. How many different Unix passwords are possible?

5. Review Exercise 1.8.28 on page 23 of the text: "You want to send postcards to 12 friends. In the shop there are only 3 kinds of postcards. In how many ways can you send the postcards, if

   (a) there is a large number of each kind of postcard, and you want to send one card to each friend;

   (b) there is a large number of each kind of postcard, and you are willing to send one or more postcards to each friend (but no one should get two identical cards);

   (c) the shop has only 4 of each kind of postcard, and you want to send one card to each friend?"